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THE

STEAM ENGINE INDICATOR.

Directions for the selection, care and use of the instrument and the analysis and computation of the diagram.

COMPILED FROM THE REGULAR ISSUES OF

POWER

With revisions and extensions comprising numerous tables.

NEW YORK:
THE POWER PUBLISHING COMPANY,
WORLD BUILDING.
1898.

PREFACE.

THE steam engine indicator has become at once the tool of a trade and the instrument of a science. The operating engineer employs it to perfect the adjustment of valves and to measure power, the physicist to investigate thermodynamic transfers and to trace the cycle of the heat engine. It is to steam engineering at once the commercial scale and the chemical balance.

The following contributions to the literature of the instrument and its diagrams have been prepared from time to time by the writer for the columns of POWER and are addressed to the practical man who desires to apply the indicator as an instrument of ordinary precision to the problems of steam engine design and operation.

F. R. LOW,
Editor POWER.

ERRATA.

Page 132 Line 6 for table VI, read table VII.

Page 133 Formula (1) for $\frac{13700}{M.E.P.}$, read $\frac{13750}{M.E.P.}$

Page 144 3rd. Line from bottom, for table VII read table VIII.

Page 145 Line 1, for table VIII read table VI.

Page 146, heading of table for 13740 W read 13750 W.

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THE STEAM ENGINE INDICATOR.

CHAPTER I.

Selection and Care of the Instrument.

There are at this writing nine or ten different steam engine indicators upon the market. As a guide to its readers in determining which of these is best suited to their purpose, it shall be the province of this work only to specify the requirements of a perfect instrument, point out the possible sources of error in the instrument as made, detail the methods of testing for such faults, and leave the reader to purchase the degree of accuracy necessary for his purpose at the lowest available price.

For certain classes of work, such as the ordinary setting of valves, the measurement of horse-power for purposes of daily record in factory work, etc., extreme accuracy is not essential. A man does not buy a chemist's balance to weigh sugar, nor an expensive chronometer for a kitchen clock. An instrument which is ordinarily correct will answer many purposes to which an indicator may be advantageously applied, and its inherent errors will probably be less than those of manipulation and observation.

For other classes of work, however, the utmost attainable precision must be insisted upon, and the very best instruments made are not good enough. In the 72-inch low pressure cylinder of the cruiser "Brooklyn" there will be developed over 100 horse-power per pound of mean effective pressure. The variation of one one-hundredth of an inch in the mean height of a diagram from one end of this cylinder would mean, with a 10-pound spring, a difference of over five horse-power in the result. If

this vessel had been built as others have with a bonus or forfeit of one hundred dollars per horse-power above or below that called for in the contract, the money involved in its exact determination would warrant the extreme of expense and pains in securing the utmost attainable precision in the measuring instruments.

In a perfect indicator the pencil should, by its vertical position on the diagram, represent exactly the pressure beneath the indicator piston at any instant; and by its horizontal position, the point which the piston has reached in its stroke at the same instant. This simple condition is impossible of attainment in practice, from the fact that the materials with which we must deal have mass. As soon as we put them into motion we have momentum to carry both the pencil and the drum away from the point to which they would have been carried by the pressure and reducing motion alone; and their inertia to prevent their instantaneous response to a change in conditions.

LIGHTNESS.—It may, therefore, be concluded that, other things being equal, that instrument will give the best results in which the least weight is moved through the least distance for the production of cards of equal size, assuming always that enough material is used to give the necessary strength and rigidity.

FREEDOM FROM FRICTION is a quality that an indicator should possess in the greatest possible degree. Detach the piston and see that the pencil levers will drop freely and without any suspicion of a catch from any position within the working range of the instrument. With the piston attached, but without any spring, raise the piston by taking delicately hold of the pencil, and work the pencil lever up and down through the full limit of its motion, feeling carefully for any interruption to its movement. Then raising the pencil nearly to the top of the paper-drum, cover the hole through which steam is admitted to the indicator with the thumb, as in Fig. 1. The pencil should sink slowly through the whole range of its motion, but should drop instantly from any point upon the removal of the thumb.

Do not get the piston too tight, through fear of its leaking. It has a whole boilerfull of steam behind it part of the time, and a large volume always, and no noticeable difference in pressure

will result from any leakage which can take place unless the leakage is so excessive as to increase the pressure on top of the piston. On condensing engines the vacuum, as indicated by the indicator, may be materially reduced if the piston is too loose, and it is unpleasant and uncleanly to have too much steam and water leaking and spattering about the instrument. The piston which will sustain the test shown in Fig. 1 will be found tight enough without excessive friction.

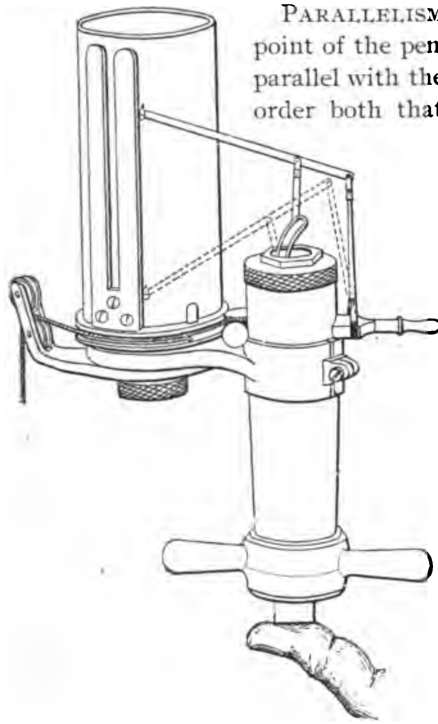


Fig. 1.

PARALLELISM.—The line in which the point of the pencil moves should be exactly parallel with the axis of the paper-drum, in order both that the pencil may bear upon the paper equally in all portions of its stroke, and that its vertical movement may be at right angles with the horizontal movement of the paper. With the piston attached but with no spring adjust the stop so that you can just see daylight between the point of the pencil and the paper on the drum. Then raise the pencil slowly through its full range by pushing the piston, and notice if the pencil point keeps the same distance from the paper. If it does not, either the spindle of the barrel is out of line with the indicator cylinder, or the pencil motion is out of line. Still sighting between the pencil and the paper, rotate the barrel by drawing out the cord. If the paper touches the pencil, or moves away from it, the drum is out of shape or improperly centered. Now, allowing the pencil to touch the paper, push the piston upward, drawing a fine vertical line upon the card; then, rotate the barrel, and draw a fine horizontal line. These lines should be perfectly straight throughout

their length, and exactly at right angles with each other, a condition which may be tested with the triangles after the card is removed from the paper-drum as shown in Fig. 2.

If the lines do not comply with these conditions, the natural inference will be that the pencil movement is incorrect, although the horizontal line may be thrown out by any vertical movement of the cylinder upon its spindle.

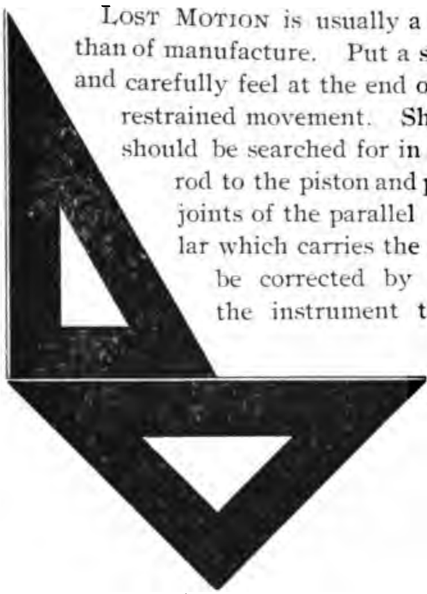


Fig. 2.

Lost Motion is usually a matter more of adjustment than of manufacture. Put a stiff spring into the indicator, and carefully feel at the end of the pencil lever for any unrestrained movement. Should such be found, its cause should be searched for in the connection of the piston rod to the piston and pencil motion, through all the joints of the parallel motion, in the fit of the collar which carries the mechanism, and if it cannot be corrected by adjustment without making the instrument too stiff to comply with the friction test above described, the instrument should be rejected.

Proportional Movement.—The movement of the pencil should be exactly proportional to that of the piston. This is an important requirement, but more difficult of test. A screw of perfectly uniform pitch should be arranged to communicate its movement to the indicator piston. With a little ingenuity a micrometer caliper can be adapted to this purpose. Turn the screw up until it has a firm bearing against the piston, then apply the pencil of the indicator to the paper and make a line by moving the drum. Then turn the screw through a number of exactly equal distances, repeating the marking process each time. The piston having been moved through an equal space after each marking, the spaces between the lines upon the paper should be equal. Great care must be taken in arranging and manipulating this test. The pencil movement is from four to six times that of the piston, and any failure to move the piston through exactly equal spaces will introduce apparent errors which will be magnified upon the card.

Count the spaces between the lines which you have drawn, then countoff the same number of spaces upon an equally divided scale of such magnitude that the aggregate length of the given number of spaces on the scale will not be less than the distance between the outside lines upon the paper. Then lay the scale across the pencil lines, as shown by Fig. 3, in such a way that the number of spaces laid off on the scale will just reach from the top to the bottom line on the diagram. For example, in the diagram shown in Fig. 3 there are 25 spaces. A "ten to the inch" scale is laid diagonally across with its zero and 25 lines upon the outside lines of the diagram. If the lines of the diagram are equally spaced they will coincide with the divisions of the scale, as in Fig. 3. If the multiplying motion of the indicator is incorrect the spaces of the diagram will be unequal, and their inequality will be apparent by their failure to meet the divisions of the scale, as in Fig. 4.

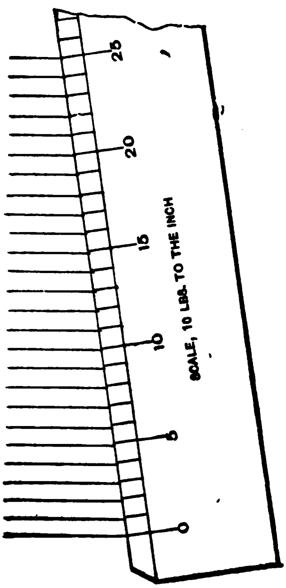


Fig. 4.

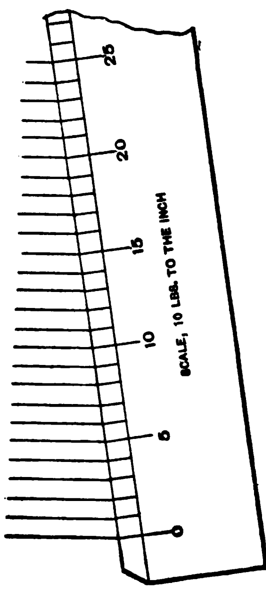


Fig. 3.

THE SPRING is the actual measuring factor of the indicator, and the apparatus required for its testing is too complicated and extensive to be at the command of the average purchaser. The test ought to be made under as nearly as possible the conditions of use,—i.e., under steam pressure, so that all the factors of temperature, etc., will be present. Most of the manufacturers

will make such tests of springs for purchasers, and the diagrams of the test may be kept as a record of the degree of accuracy of of the instrument at that time. It is well also to have such tests made occasionally after the instrument has been in use, and especially just before and after applying it to work of particular importance. The test consists of appplying steam to the indicator piston at pressures increasing by equal amounts, say, for ordinary springs, five pounds. As each five pounds is reached a line is drawn upon the card, a standard gage or, better, a mercury column being used to indicate the pressures. (An elaborate apparatus for drawing these lines automatically as the mercury rises in the column is maintained by the United States Government at the New York Navy Yard, for testing and standardizing the indicators used by the navy.) The pressure is then allowed to fall, and marks are again made as the gage passes the points which were noted in the upward series. If the spring and all the transmitting and recording mechanism were perfect, and the indicator without friction, the spaces for equal changes in pressure would be of equal width, and the lines indicating the same pressures would be coincident, whether drawn when the piston was going up or coming down. This degree of perfection is rarely if ever reached, for even if the spring compresses equal distances for equal increments of pressure throughout its entire range, and its movement is transmitted correctly to the pencil, the friction of the piston, of the pencil movement, and of the pencil on the paper all combine in opposing the motion of the piston in both directions, so that the lines of the upward series are too low and those of the downward series too high by an amount equivalent to the frictional resistance upon the scale of the spring. As a consequence, diagrams such as Fig. 6 are not uncommon, while Fig. 5 is accounted ordinarily good.

The above qualities are necessary to an indicator for accuracy. Other points, more in the nature of conveniences than essentials, but which may be well considered in selecting an instrument, are the comparative simplicity of changing springs, adjustment for height of atmospheric line, changing from right to left hand and *vice versa*, adjusting the drum-spring and leading pulley, attaching the indicator to the cock, etc. For holding the lead, the end of the pencil lever in some indicators is formed into a

light steel quill of a size which will hold the lead firmly when forced through it. In other makes the end of the pencil lever is reinforced and threaded internally, the lead being screwed through it. The preference of the writer is decidedly for the first method. The quill being split lengthwise adapts itself by its elasticity to varying sizes of lead, and may be closed with a pair of pincers if it fails to close upon a lead of small diameter

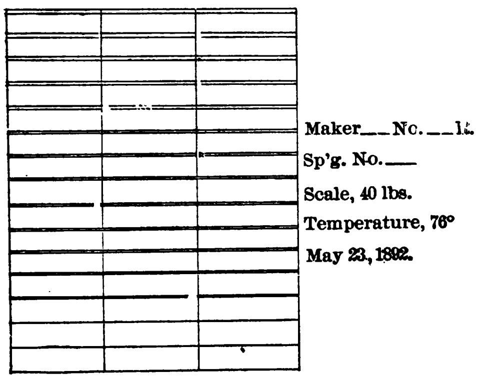


Fig. 5.

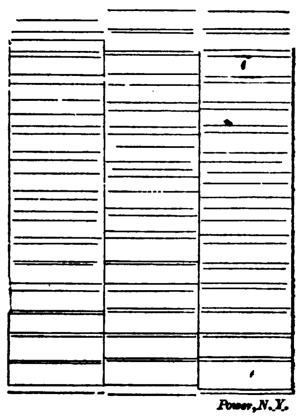


Fig. 6.

after being used with a larger size. As the point is shortened by resharpening, the lead can be pushed forward, and if it breaks short off it is easily pushed out of the holder with a match or toothpick. The threaded end is adapted to only one size of lead, and with the short bearing afforded it is apt to get loose and wabble. If it breaks off short, it must be dug out of the threaded portion; and if the threaded method offers any compensating advantages the author has yet to learn of them.

So far as springs are concerned, if the use of the instrument is to be confined to one's own plant it is easy to select a spring or set of springs adapted to the pressures and speeds to be encountered. If the instrument is to be used promiscuously, the more springs the operator can own the better will he be equipped to meet the conditions of practice. In selecting a spring, aim to get as large a diagram as possible without undue distortion. If a diagram be taken with a 20 spring an error of measurement of one one-hundredth of an inch would influence the results only one-fifth of a pound. With a 50 spring the same error in

measurement would represent a departure of one-half pound. Or since the average useful pressure upon which the power indicated by the diagram depends is proportional to the area of the diagram, consider a diagram taken with a 20 spring having an average height of 2 inches and a length of 4 inches as compared with one taken from the same cylinder with a 40 spring and a length of 2 inches. The area of the first diagram would be 8 inches, of the second 2 inches, and the average useful or "mean effective pressure" of course 40 in both cases.

$$\frac{(\text{Area})\ 8 \times 20\ (\text{scale})}{(\text{length})\ 4} = 40. \qquad \frac{(\text{Area})\ 2 \times 40\ (\text{scale})}{(\text{length})\ 2} = 40.$$

In the large diagram 40 pounds of pressure are represented by 8 inches of area, or 5 pounds to an inch, and an error in measurement of the area of one one-hundredth of a square inch would involve an error of but five one-hundredths of a pound in the indicated pressure. In the case of the smaller diagram 40 pounds pressure is represented by 2 square inches of area, 20 pounds to the inch, and a deviation of one one-hundredth of a square inch from the truth in measuring this area will involve an error of two-tenths of a pound.

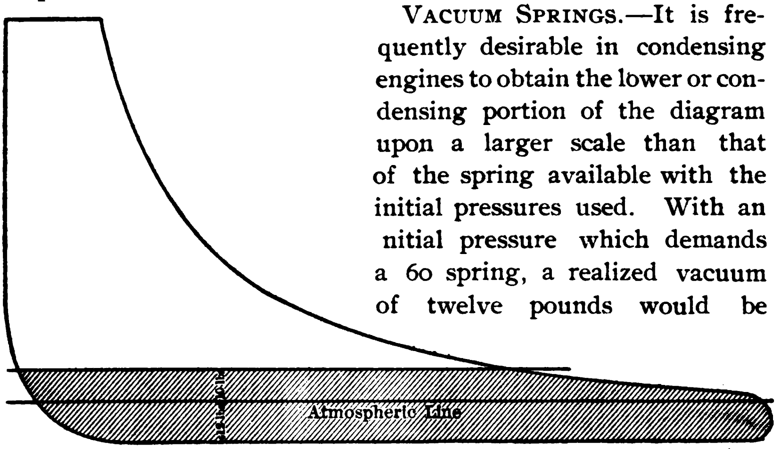
It is therefore advisable to have the area as large as possible *and have it right.*

On the other hand, the allowable movement of both the pencil and the drum is limited by the effects of momentum. At high speeds a light spring and long movement of the drum would result in a diagram so distorted by the effects of momentum and inertia as to introduce errors much more serious than those which are likely to occur from inaccurate measurement of a smaller and more perfect diagram. The speed as well as the pressure will therefore have a bearing upon the spring selected, and will also influence the selection as between the standard size of paper drum which is used for moderate speeds, and the smaller drums which some of the makers supply for high speed work. Some manufacturers furnish two sizes of drums, which may be used interchangeably upon the same instrument, adapting it to the highest and lowest speeds.

In some instruments the position of the atmospheric line is fixed, in others it is adjustable, so that in indicating a non-condensing engine the base line may be lowered and the whole of

the allowable movement of the pencil utilized for the height of the diagram. The springs made by American manufactures are usually scaled decimally, that is, 10, 20, 30, 40, etc., pounds to the inch, and the ***maximum*** pressure to which each spring is adapted as recommended by their several makers is as follows:

VACUUM SPRINGS.—It is frequently desirable in condensing engines to obtain the lower or condensing portion of the diagram upon a larger scale than that of the spring available with the initial pressures used. With an nitial pressure which demands a 60 spring, a realized vacuum of twelve pounds would be



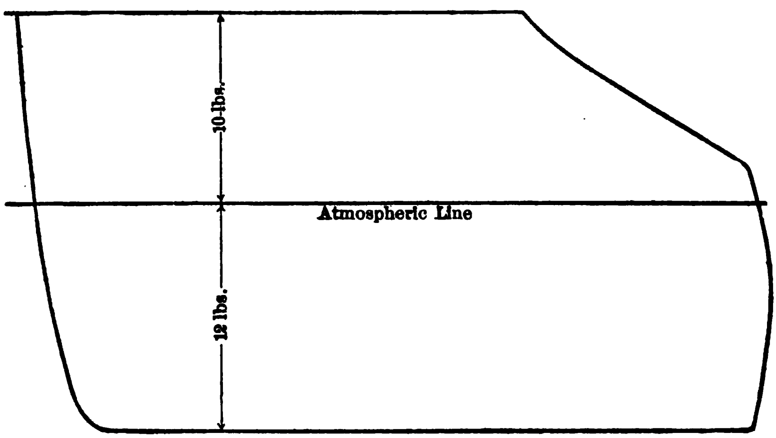


Fig. 7.

represented by a line only one-fifth of an inch below the atmospheric line, Fig. 7, giving a very small area to the condenser portion of the diagram. In order to obtain this area upon a larger scale, giving increased accuracy of measurement,

showing more clearly the points of release and compression, etc., springs of low tension are sometimes fitted with bosses or studs, which prevent their closing beyond a certain point, while they are free to extend to any amount.

In Fig. 7 are shown two diagrams, the first drawn to a 60 scale; and beneath it the shaded portion of the diagram is shown expanded to a 10 scale. Notice how much more prominently the points of release and compression are shown, on account of the more rapid vertical movement with the same horizontal movement; and how much less an error of a few hundredths of a square inch in measuring the area of the condensing portion of the card would affect the result. A spring made especially for this purpose by the American Steam Gauge Co. is shown in Fig. 8. It is wound very closely, and the coils close upon themselves before the pencil movement can attain a dangerous amount of motion. The large number of coils lying in so nearly a horizontal direction admits of sufficient elasticity with a good sized wire, while there is uniformity of movement throughout the desired range. Inasmuch as there has been found to be a difference in the action of springs when compressed and extended, these springs are scaled for extension only.

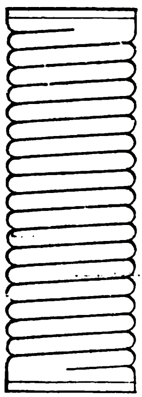


Fig. 8.

SCALES.—For a measuring scale, the author uses a six inch engineer's rule, triangular in cross section as shown in Fig. 9, and ~~graduated upon~~ its six edges to 10ths, 20ths, 30ths, 40ths, 50ths, and 60ths of an inch. This rule not only furnishes the six scales mentioned in one rule, but by estimating half spaces we can measure an 80 with a 40 scale and 100 with the fifty. With the lower scales, where the distances are greater~~, we can measure accurately~~ to half pounds by using the 60 scale for a 30 spring or the 40 for a 20, etc. The 50 scale is also very useful for measuring the length of the diagram, each division representing .02 of an inch, and the length of 6 inches being more than sufficient for any diagram.

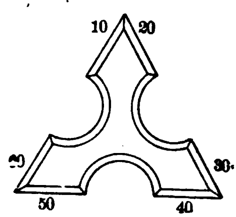


Fig. 9.

DUPLICATE PARTS.—Much annoyance and loss of time may be saved by carrying in the indicator box duplicates of those parts liable to loss or derangement. An additional drum spring, and two or three of the smaller screws which have to be frequently removed in changing springs, etc., and which are liable to disappear down a crack or somewhere else when most wanted, will allow a test to proceed smoothly, when its interruption would be particularly annoying from the insignificance of its cause.

LEADS.—Select a hard lead of good smooth quality and of small diameter, and use but a small piece at a time. At the end of the pencil lever, where the motion is greatest, the weight should be reduced to the smallest possible value. If pointed with a fine file and rubbed down with an emery stick, such as is used for sharpening draftsmen's pencils, or a fine stone, it will wear longer and be smoother and more satisfactory than if whittled into shape. A little metallic case of such leads already pointed is a very convenient portion of an outfit.

LUBRICATION.—For lubricating the bearings of the instrument a light machinery oil, one which will not gum or corrode, should be used. A small vial of such oil usually accompanies the instrument, some makers furnishing porpoise oil, such as is used for clocks and watches. The piston, however, is better lubricated with cylinder oil, and the small cans which are furnished for bicyclists' use, and which fit readily into the tray of the indicator box, furnish a convenient means of carrying a filtered supply in a form readily available for cleanly use. The manufacturer's filtering should not be accepted. Filter the oil carefully yourself, and see that the can is perfectly clean. A small particle of grit upon the piston of an indicator will not only throw the diagram into the most unaccountable contortions, but may scratch and injure both cylinder and piston to a serious degree.

PAPER.—Use hard, tough, smoothly calendered paper of a width sufficient to include the highest allowable pencil travel and about an inch longer than the circumference of the barrel. Such paper can be procured cut to any desired size, of almost any printer.

CHAPTER II.

REDUCING MOTION.

In order to use the indicator, a means must be provided for moving the paper drum exactly in time with the engine piston. This movement is usually derived from the cross-head, and the appliance used to reduce the movement to that adapted to the paper-barrel is spoken of as the "reducing motion."

THE PENDULUM LEVER.—The most primitive expedient for this purpose is a lever suspended from the ceiling or other suitable support, and connected at its lower end with the cross-head in such a way that it will be swung back and forth as the engine makes its revolutions, as in Fig. 10. The motion of the lever increases from nothing at the point of suspension to approximately the full stroke of the engine at the cross-head end, the amount of motion being directly proportional to the distance from the point of suspension. A point midway of the lever would have a motion equal to one-half the stroke; one-quarter of the way from the point of suspension, one-quarter stroke, etc.

Letting $l =$ distance between pivot and cord pin,
$L =$ length of lever,
$s =$ desired length of diagram;
$S =$ stroke of engine,

then the diagram will be $\frac{l}{L}$ ths of the stroke, and the cord must be attached at a point $\frac{s}{S}$ ths of the total length of the lever from the point of suspension. For

$$l : L :: s : S$$

that is; as the distance between the pivot and the point to which the cord is attached is to the total length of the lever, so is the motion at that point and the length of the diagram to be derived from that motion, to the stroke of the engine.

$$\frac{l}{L}=\frac{s}{S} \qquad \text{and } l=\frac{L\,s}{S} \qquad \text{and } s=\frac{l\,S}{L},$$

TO FIND THE POINT OF ATTACHMENT, or the distance from the point of suspension at which the cord should be attached to produce a given length of diagram:

RULE:—*Multiply the total length of the lever by the desired length of diagram, and divide by the stroke of the engine, all in inches.*

EXAMPLE:—With a lever 60 inches in length on an engine of

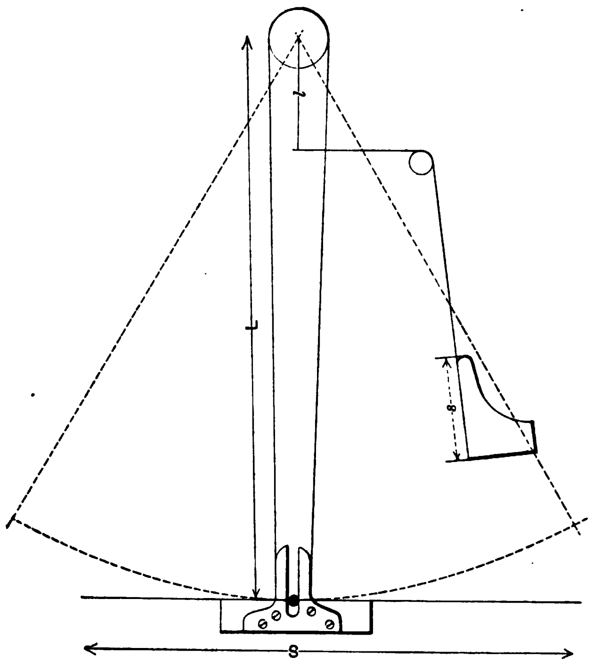


Fig. 10.

24-inch stroke, how far would you attach the cord from the point of suspension to produce a diagram 4 inches in length?

$$\text{Operation: } \frac{60 \times 4}{24} = 10 \text{ inches.}$$

TO FIND THE LENGTH OF DIAGRAM produced by a cord at a given point of attachment;

RULE:—*Multiply the distance from the pivot to the point of attachment by the stroke of the engine, aud divide by the total length of the lever, all in inches.*

EXAMPLE:—What length of diagram would be produced by attaching the cord $4\frac{1}{2}$ inches from the pivot on a lever 20 inches in length attached to a cross-head having a stroke of 12 inches?

$$\text{Operation: } \frac{4.5 \times 12}{20} = 2.7 \text{ inches.}$$

The total length of the lever is measured from the point of suspension to the point of attachment to the cross-head, and is variable in some of the arrangements to be shown. As the variation bears a small proportion to the total length, and the length of diagram is usually figured only to keep within the limits of the paper-drum, especial refinement in this particular is unnecessary. In order to get the full motion of the pin, the cord must be led off in the direction of the pin's greatest

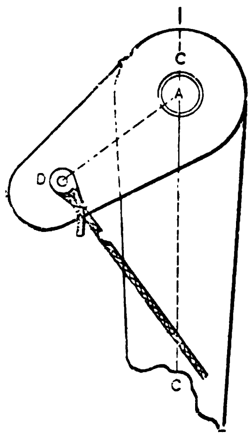


Fig. 11.

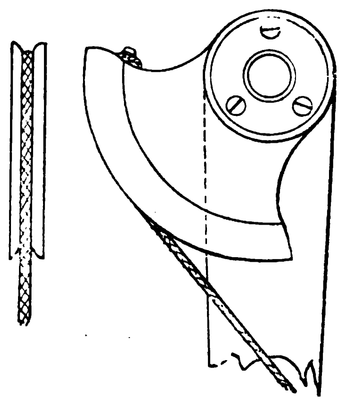


Fig. 12.

movement, i. e., at right angles to the lever when the lever is itself at right angles to the guides. It will be readily seen that if the cord were led off parallel to the lever it would receive very little motion. It is desirable to avoid the use of leading pulleys as in Fig. 10; and Figs. 11 and 12 show two methods of accomplishing this, the first by putting on a segment of a circle, called a brumbo pulley, having a radius equal to the distance l from the pivot to the point of attachment of the cord, and so placed that the cord may be led straight to the indicator without running on to the corners of the segment at the extremes of the stroke. In Fig. 12 a supplementary lever is added in such a position that when the main lever *CC* is at right angles

to the guides the line *AD* will be at right angles to the cord when the latter is led in the desired direction.

In all motions of this kind there is a radical defect due to the fact that while the cross-head moves in a straight line any point on the lever swings through the arc of a circle. In Fig. 13 let the line OX represent the stroke of an engine. A lever attached to the cross-head and suitably suspended at the other end would take, as the stroke progressed, the positions 1 1′, 2 2′, 3 3′, etc., and a pin attached to the lever at 1′ would move through the arc shown. Divide the stroke into 8 equal parts, as indicated

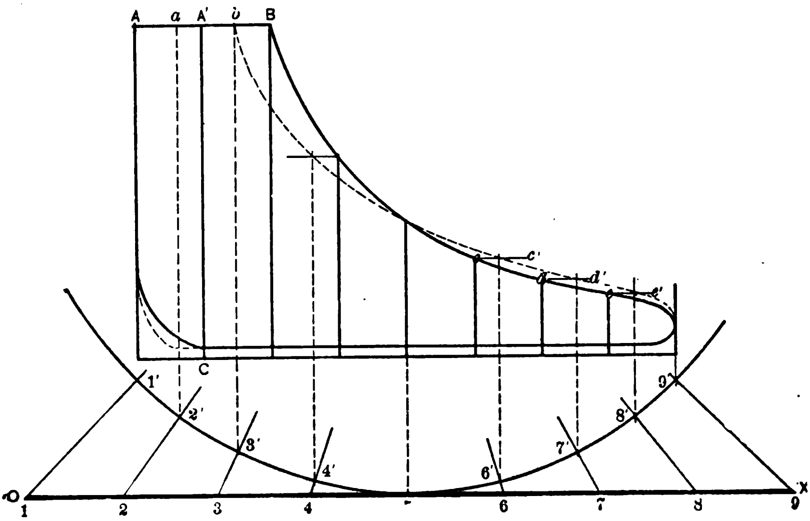


Fig. 13.

by the numbered divisions, and as the cross-head completes each division of the stroke the position of the pin will be indicated by the corresponding number upon the arc. The length of the diagram will be the horizontal distance between 1′ and 9′, but the distribution of motion between these points will not be equal for equal movements of the cross-head. When the cross-head moves from 1 to 2, one-eighth of the stroke, the pin will move from 1′ to 2′, and the card will be moved only through a distance *Aa* instead of through *AA′* one-eighth of its own length; and for each division of the stroke the proper division of the diagram is indicated by the full lines, and the division that

would be derived from the motion of the pin by the dotted lines. Supposing the cut-off to take place at a quarter of the stroke, this point should be at B, but would appear at b, and the dotted and incorrect instead of the full-line correct diagram would be drawn. The points coincide in the middle of the diagram, and become as much too late at the end as they were too early at the beginning, the points which should be at c, d, and e being at c', d', and e' respectively. The distortion shown here is exaggerated

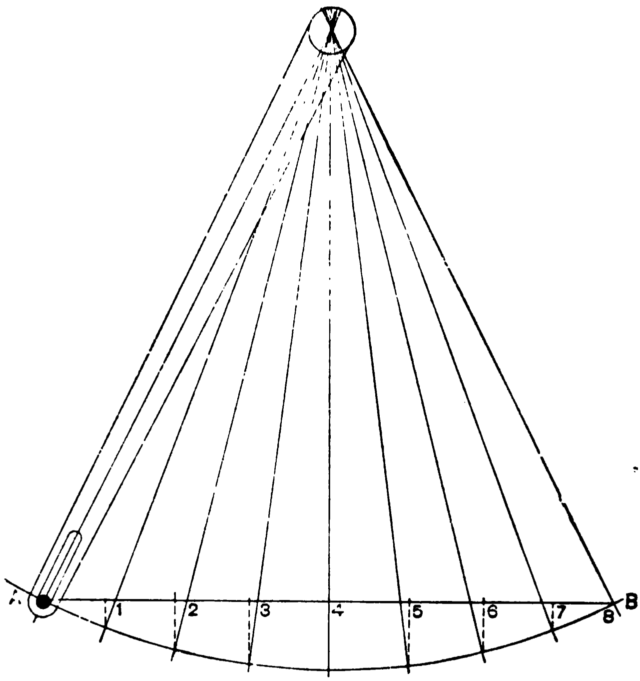


Fig. 14.

on account of the shortness of the lever. It decreases as the length of the lever in proportion to the stroke is increased, and for this reason it is advisable never to use a lever less than one and a half times the length of the stroke. The point of suspension of the lever should be directly over its point of attachment to the cross-head when the latter is in the center of its stroke.

The amount of distortion varies also with the manner of attachment to the cross-head. Fig. 14 represents a slotted lever working over a pin in the cross-head. As each eighth of the

stroke is completed the lever will occupy the positions shown by the lines passing from the point of suspension through the corresponding divisions, and the straight motion, as *AB*, to be derived from any point upon the lever will be unequally divided, as shown by the intersections of the dotted lines. Fig. 15 represents a lever fitted with a pin, which is carried by a slot in the cross-head. As the cross-head and the slot move through successive eighths of the stroke, the pin is carried also through equal divisions, and motion in a line *CD*, at right angles to the lever in its central position, would be equally distributed, as

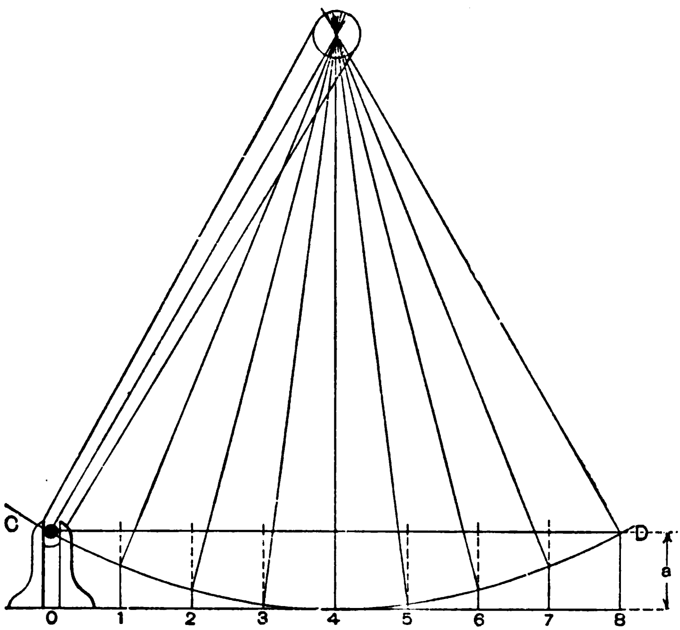


Fig. 15.

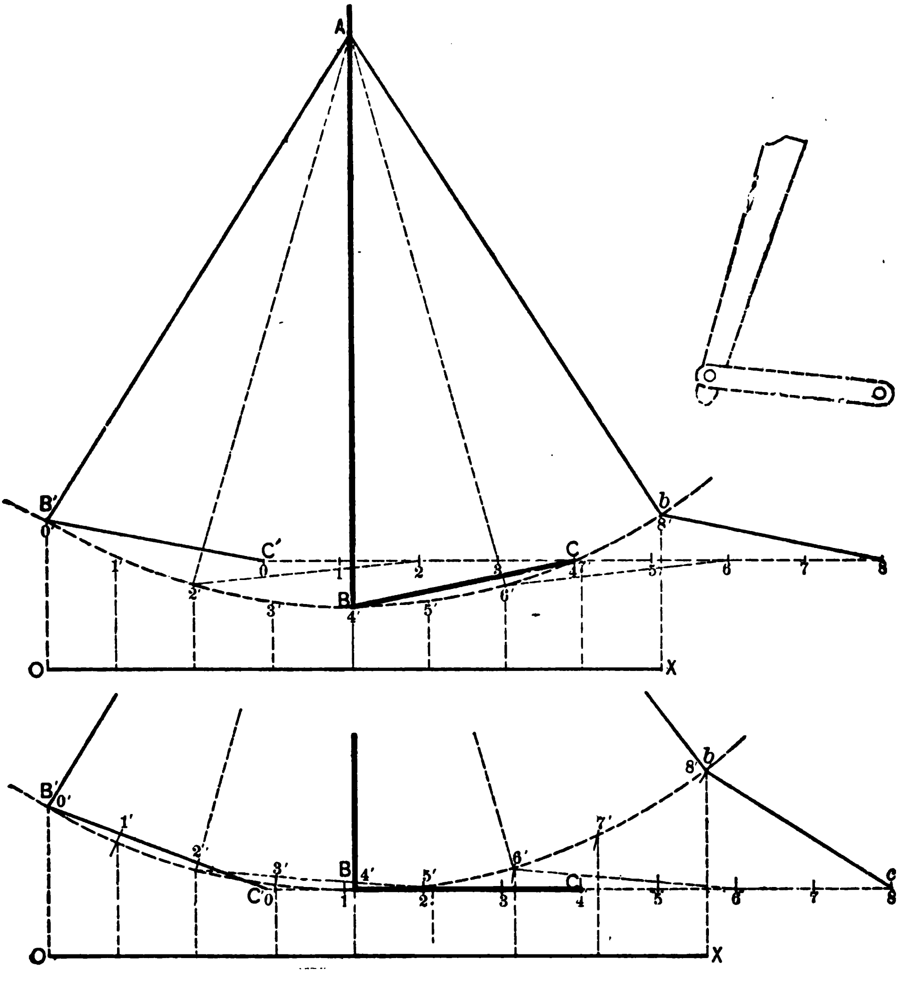
shown by the intersections of the dotted lines referring the positions of the pin for the eight equal divisions of the stroke to the line of motion *CD*. If it were not for the angular movement *a*, of the cord with which this motion is taken off, and which produces an inequality in the transmitted motion, just as a connecting rod does in the travel of the piston for equal movements of the crank, this arrangement would be perfectly accurate. The cord is usually so long, however, that its angular motion is immaterial. This feature cannot be eliminated by using the arc

or brumbo pulley, for while the latter disposes of the angular movement of the string, it gives a movement proportional to the angular motion of the lever, which is not equally divided, i. e., the lever does not move through equal arcs of a circle for equal movements of the cross-head. The use of the brumbo in this case would therefore introduce rather than eliminate an error. While this arrangement produces upon paper an almost perfectly proportional reduction of the motion, its effects in practice are not so precise. The long lever is cumbersome, the slotted guide an awkward thing to make and attach to the cross-head, and unless the pin is accurately fitted, the distortion and annoyance due to lost motion will be greater than the inherent error of simpler construction.

Instead of the slot upon the cross-head a short connection rod may be used, as in Fig. 16. In this case the end of the main lever, instead of working up and down in a vertical slot, is swung in the arc of a circle of the radius of the short connecting rod. The departure from the vertical line will be least if the levers are so attached that the vibrating end of the small lever will be as much below the path of the cross-head end when the main lever is in its central position as it is above it when in the extreme positions. This will be understood by referring to Fig. 17, in which the levers are represented by the lines *AB* and *BC*, the cross-head traveling on the line numbered 0 to 8. When the cross-head is in the middle of its stroke at 4, the ends *B* of the levers are as much below the line in which the cross-head travels as they are above it in the extreme position shown at B′ and *b*. When the cross-head in its movement arrived at the points 1, 2, 3, etc., representing equal sub-divisions of its travel, the ends of the levers would be respectively at the figures 1′, 2′, 3′, etc., crossing the line of motion of the cross-head twice during the stroke. Referring these points to the straight line, OX by the dotted lines, it will be seen that the subdivisions very nearly reproduce the equal subdivisions of the movement of the cross-head from which they are derived.

If the levers had been arranged at a right angle when in the center of the stroke, as in Fig. 18, the entire vibration of the levers would take place above the plane in which the cross-head moves. The greater distance to which the end of the small lever is carried from that plane would increase the angle between

them and introduce a greater distortion, as will be seen from Fig. 18, in which the same process has been carried out as in Fig. 16, the movement derived from any point in the main lever



Figs. 16, 17 and 18.

being represented by the subdivisions into which the dotted lines divide the line *ox*, which as will be seen, are far more irregular than in Fig. 17.

THE PANTOGRAPH.—Engravers and draftsmen have an instrument called the "pantograph" for reproducing drawings upon a different scale. One of the cheaper forms of the instrument is shown in Fig. 19. A drawing closely followed with the tracing point is reproduced upon a smaller scale by the pencil point as shown. If the tracing point draws a circle the pencil draws a smaller circle; if the tracing point draws a straight line the pencil point draws a shorter straight line, and the movement of the pencil point and tracing point are proportional throughout. When the tracing point has drawn one-tenth of *its* line the pencil has drawn one-tenth of its line and so on to completion. It will readily be seen that if the tracing point of the pantograph be attached to an engine cross-head the pencil will accurately reproduce the stroke upon a reduced scale and substituting a cord pin for the pencil we have a perfect reproduction of the motion of the cross-head for transmission to the paper-barrel. The two forms in which the pantograph is used for indicator purposes are shown in Figs. 20 and 21.

Fig. 19.

Of both forms it is true that the cord pin *C* must be directly in line with the stationary point *A* and the point of attachment to the cross-head *B*, as indicated by the dotted lines; also that the distance from the point of suspension *A* to the cord-pin *C* is to the distance between *A* and *B* as the length of the diagram is to the stroke of the engine so that the rules given for the lever will apply equally well to the pantograph. The distance *AC* may be varied by moving the strip *C*, Fig. 20, into one or another of the holes 1, 2, 3, etc., and then moving the cord pin into that hole in the strip which

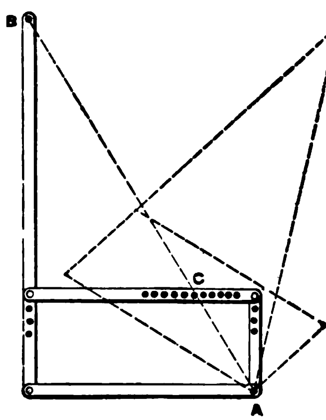


Fig. 21.

is in the center line of the instrument. The author has pasted into the cover of his indicator box Table 1, which is correct

TABLE 1—PANTOGRAPH TABLE.

Hole. No	Proportion Card to Stroke.	Decimal Fraction of Stroke	Divided by	Longest Stroke.
1	1 : 16	.0625	16.	72″
2	1 : 12	.0833	12.	54″
3	5 : 48	.1042	9.6	42″
4	1 : .8	.1250	8.	36″
5	7 : 48	.1458	6.9	31″
6	1 : 6	.1667	6.	27″
7	3 : 16	.1875	5.3	24″
8	5 : 24	.2083	4.8	22″
9	11 : 48	.2292	4.4	20″
10	1 : 4	.2500	4.	18″
11	13 : 48	.2836	3.7	16″

for the pantograph which he uses.

This shows that when the pin is in the first hole (No. 1) the diagram will be one-sixteenth or .0625 of the length of the

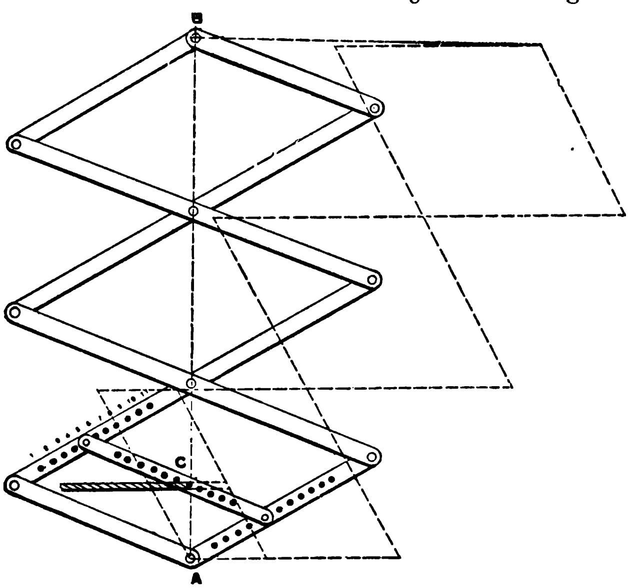


Fig. 20.

stroke; in the fifth hole seven forty-eighths, or .1458, etc. To find the movement of the cord pin at any hole with an engine of

given stroke, multiply the stroke in inches by the decimal fraction opposite the number of hole given; or divide the stroke in inches by the number in the column headed "divided by" opposite the number of the hole given.

To find the proper hole to use with an engine of given stroke to produce a diagram of a required length: Divide the length of the stroke in inches by the desired length of diagram in inches. The number nearest to the quotient in the column headed "divided by" will be opposite the number of the hole which will nearest produce that length. The ratio of the diagram to the stroke may coincide with one of those given in the table. Thus, if it was desired to produce a four-inch diagram from a thirty-two inch stroke, the ratio would be $4 : 32 = 1 : 8$, and it is apparent from the columns of proportions given that the pin in the fourth hole will have the required movement. The same result may be arrived at by dividing the length of the diagram in inches by the stroke in inches and selecting the pinhole which is opposite the nearest decimal fraction to that obtained. The last column of the table gives the longest strokes allowable for the various positions of the pin to produce diagrams not exceeding four and a half inches in length, which is about the capacity of the ordinary drum. Additional columns for other lengths may be made out if desired by multiplying the figures in the column headed "divided by" by the length of diagram desired. Such a column, for instance, might be added for the maximum length of diagram allowable with the smaller drum, although the smaller instruments are usually used upon engines of high rotative speeds, where the pantograph is not adapted as a reducing motion.

In the other form of pantograph, Fig. 21, holes are provided for different positions of the strip *C*, and other holes in *C* for bringing the cord pin in line with *A* and *B*. Other holes are sometimes provided for changing the point of attachment to the cross-head, in which case the cord pin must always be in line with the stationary point *A* and the hole which is used for the cross-head attachment, and the length of the diagram will be to the length of the stroke as *AC* is to *AB*.

In view of this latter fact, if the pantograph is opened until *AB* equals the stroke of the engine, then *AC* will be the length of the diagram at once, and with the shorter strokes this fact

may be used to advantage in setting the pantograph. Suppose we have a 24-inch stroke. Open the pantograph until a two-foot rule will just extend from center to center of pins *A* and *B*, as in Fig. 20, then the distance *C* to *A* will be the length of diagram to be expected, and the pin may be so adjusted as to make this distance equal to the length of diagram desired. For greater lengths of stroke this principle may still be used by halving. Take a 72-inch stroke, for instance. One-half of this is three feet. Open the pantograph to three feet, then the distance *AC* will equal one-half the length of the diagram.

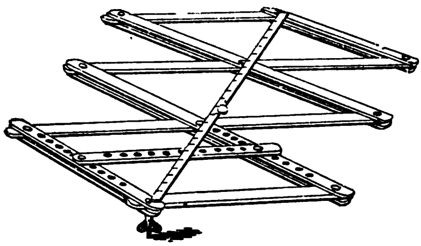


Fig. 22.

There is no patent upon the pantograph in either of these forms, and anybody who has tools and knows how to use them can make one for himself. The members are usually made of strips of hard wood one and one-eighth by five-sixteenths of an inch, and sixteen inches between the pivoted points. These strips are put together in the manner indicated in the illustration, the single strips running between the double, making it stiff and substantial. The levers must work perfectly easy, and all lost motion be avoided. The joints must be well made and the pivot holes should be bushed. A good form of joint, designed by Mr. E. K. Conover of Newark, N. J., is shown in Fig. 23. It allows for taking up lost motion by filing off the bush, and permits the bearing to be taken apart and oiled occasionally. The holes which are used for the different positions of the strip and of the cord pin are usually tapped directly into the wood, but the tops are apt to be forced out or the threads crossed and cut, and a better arrangement would be to insert strips of brass at these places, and drill and tap the holes into them.

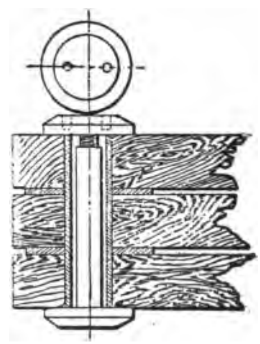


Fig. 23.

So far as the correctness of the reduction goes it makes no

difference where the stationary end of the pantograph is placed. We have seen engineers measure with a great deal of care to locate this point accurately in the center of the stroke, knowing probably that this had to be done for the lever and assuming that the pantagraph required similar arrangement. The cord, of

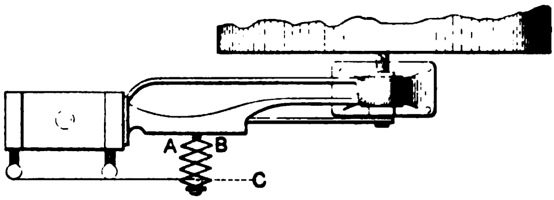


Fig. 24.

course, should be led off in the line of motion of the pin, i.e., parallel to the guides, and, since it is desirable to dispense with the use of leading pulleys, when the pantograph is used horizontally as in Fig. 24, the post should be placed at such a distance from the guides and at such a height as will bring the cord pin directly in line with the indicators, so that the cord can be led direct as shown in the plan. The point to be looked out for is that the corners *A* and *B* of the pantograph do not come in contact with the guides at the extremes of the stroke. We have seen several good pantographs spoiled in that way, and plead guilty to one wreck ourselves from that cause. Now we try it by having the engine turned over, if this can be done easily, while holding the stationary end of the pantograph, moving it, if it hits, into a position in which it will clear; or if the engine is a large one, by locating the extreme points of the pantograph's travel by measurement, and carrying the cross-head end through the range so determined in as nearly as possible the line that it will travel, observing that it clears throughout the stroke. When the pantograph is all attached and running, place your eye at *C*

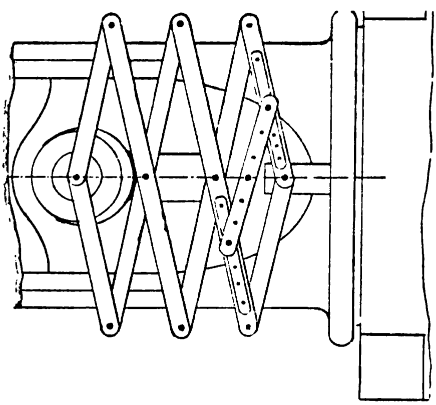


Fig. 25.

and sight the cord pin. It should move in a straight line to and from your eye. If it has any sidewise motion something is wrong; probably the pin is not in the center line of the instrument. The stationary post will come about in the middle of the guide, with this arrangement, as if moved much to either end it will bring the corner at that end in contact.

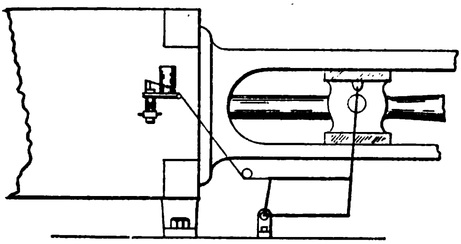


Fig. 26.

Remembering that it makes no difference how the pantograph is set, horizontally, perpendicularly, or obliquely; so long as it will clear, we may place it in any position to favor leading the cord to the indicators. Fig. 25 shows how it may be used on an engine whose stroke does not exceed the length to which the pantograph may be easily opened. The other form of pantograph may be attached to the floor, as in Fig. 26, in which case a leading pulley is required, but where the stroke of the engine will allow it had better be attached as in Figs. 27 and 28.

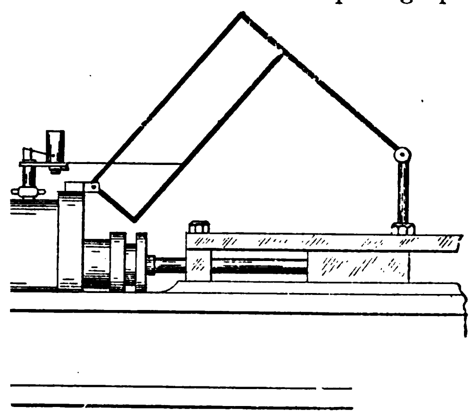


Fig. 27.

Fig. 29, from the catalogue of the Buckeye Engine Co., shows an adaptation of the pantograph for that engine. The cord is attached to the end of a short bar which slides freely in a bearing in the carrying post. This bar is connected to the lever *CD* by means of a short link *AB*. The lever is connected to a stud attached to the cross-head at *E* by the bar *DE*.

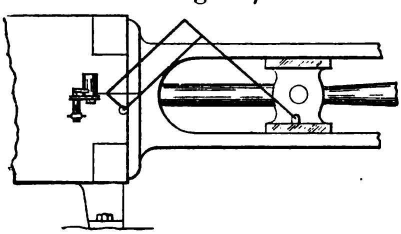


Fig. 28.

The proportions of the parts are such that the points *CBE* are in a straight line at all times, and this being the case the distortions of the movement of the lever due to the vibration of the link *DE*

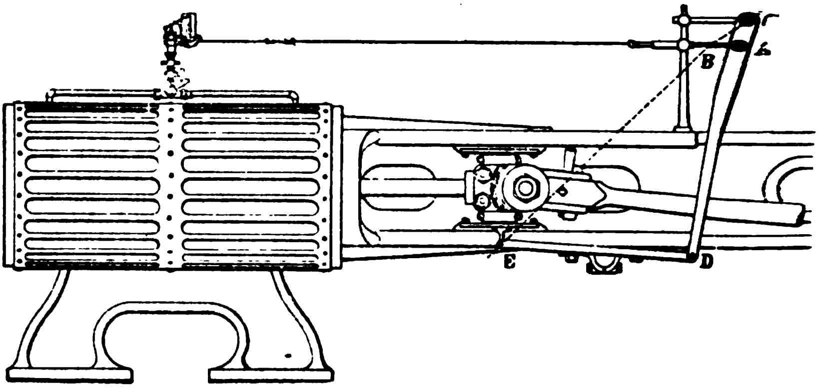


Fig. 29.

will be exactly corrected by the equal vibration of the short link. This makes a good rig for a permanent fixture, but must be proportioned for the engine upon which it is used, as it cannot, except within very narrow limits, be adjusted for engines of different sizes. The cord must, of course, be led off in the line of motion of the short bar.

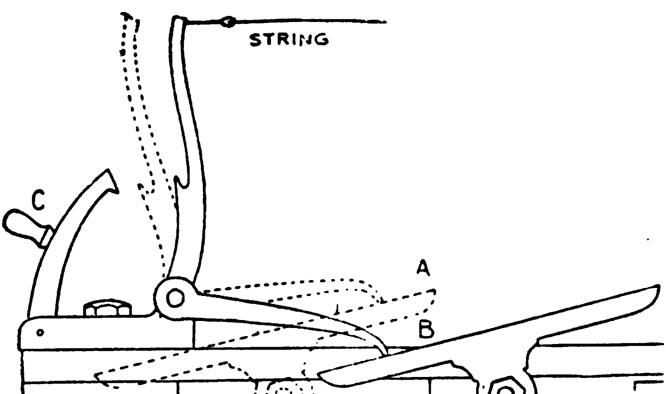


Fig. 30.

Fig. 30 shows a very good motion for short strokes. The amount of motion given to the bell crank may be varied by

changing the inclination of the plane which is attached to the cross-head, and the vertical arm may be of such length as to bring the cord in line with the indicator. The catch *C* holds the foot up off the plane and stops the instrument without unhooking the cord or leaving it flapping as with a detent on the indicator drum.

This chapter would be incomplete without some mention of the reducing wheels which have recently appeared upon the market in several improved forms. Fig. 31 shows a home-made

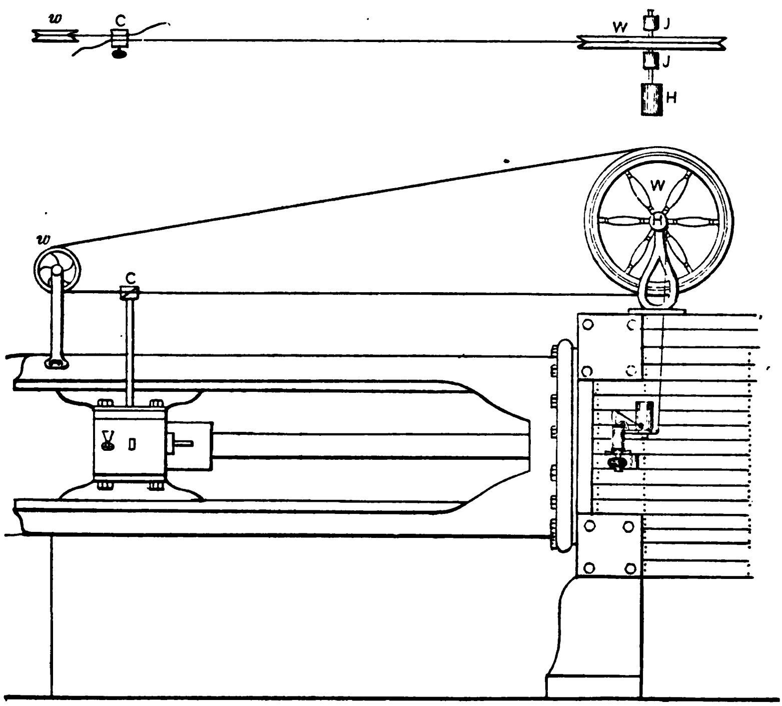


Fig. 31.

form. A standard upon the cross-head is clamped at *c* to a cord which passes around the pulleys *W* and *w*, the hub H, from which motion is taken to the indicator, bearing the same proportion to the wheel *W* that the length of diagram is to bear to the stroke. This arrangement has the advantage that the wheel *W* is kept in time with the piston by being held from overturning through momentum by the cord. Another cord can

be led from *H* to the indicator upon the back end of the cylinder. The trouble with those reducing wheels which are pulled out by the cord and returned by means of a spring has been that having considerable mass they acquired a momentum which carried them after the cross-head had stopped pulling, and distorted the stroke, like a heavy paper barrel with a weak drum spring on an

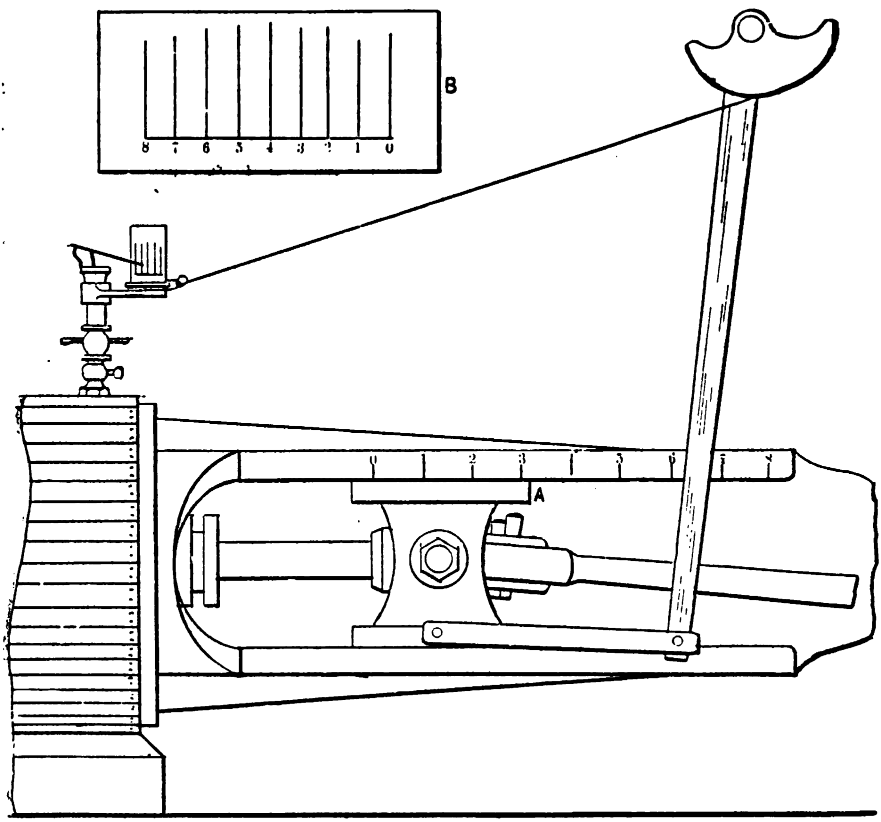


Fig. 32.

indicator at high speed. Several forms of reducing wheels have recently been put upon the market, however, in which lightness of material and construction have combined to form a device which is not only handy in application to different sizes and kinds of engines but reasonably accurate at considerable speeds.

Finally, whatever form of motion is used, there are two tests

which should be tried. The first of these is shown in Fig. 32, where the stroke 0-8. of the cross-head is divided into 8 equal parts. With the reducing motion attached to the indicator, put the engine on the center, the corner *A* of the cross-head being at zero. In this position make a vertical mark upon the indicator card by raising the pencil lever. Then move the cross-head successively to 1, 2, 3, etc., at each point making a mark upon the card. If the diagram is found to be equally spaced your motion is correct so far as the reduction is concerned. Now give the engine steam, and while it is turning over slowly apply the pencil, and hold it on during a complete revolution, making an "atmospheric" line. Raise the pencil about a sixteenth of an inch, let the engine get up to speed, and draw another line in the same way. If there is a considerable difference in the length the diagram will be distorted by the momentum of your reducing motion, or of the paper drum of the indicator itself, or by the stretching of your cord. The most that you can do is to take up all lost motion, use short cord or wire and adjust your drum spring to get the least possible discrepancy.

CHAPTER III.

APPLICATION.

Having selected our instrument and laid out an appropriate reducing motion, we are prepared to consider the attachment of the indicator to the cylinder and the method of its manipulation.

Most of the first-class engines of recent build are sent out of the shop with the cylinder drilled and tapped for the application

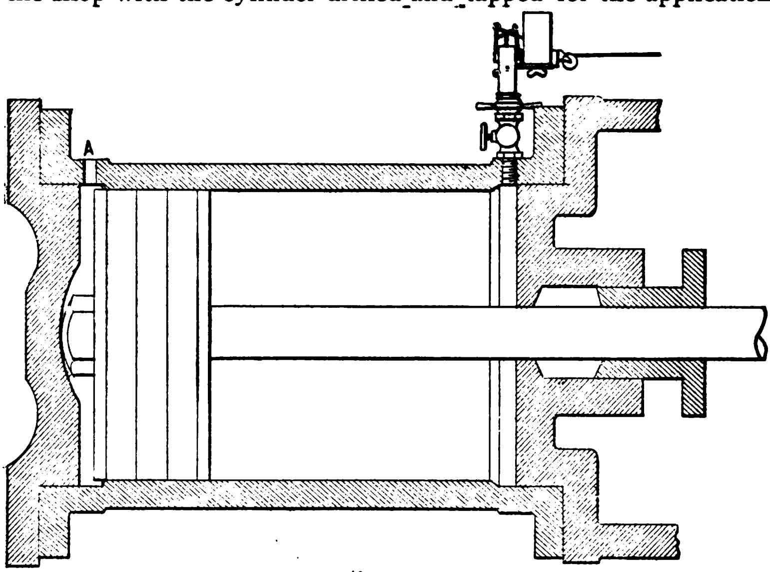


Fig. 33.

of the indicator, and plugged holes for this purpose will be found in the side or top of the cylinder by removing the lagging. When a cylinder is not tapped, the two points to be considered in locating the point for drilling are first, to so place the hole that throughout the stroke there shall be a constant uninterrupted communication between the cylinders of the indicator and the

engine; and secondly, to so locate the instrument as to lead off from it most conveniently to the reducing motion.

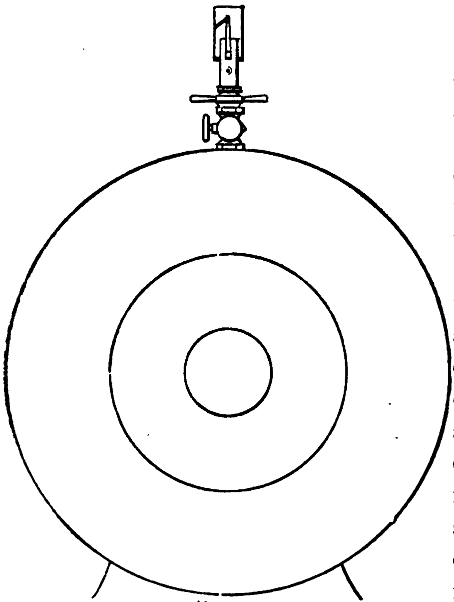


Fig. 34.

The first object is most readily attained by tapping directly into the heads, and as this is rather a more simple process for the machinist than tapping into the counter-bore, especially when room is limited, it is frequently done. Except in a few instances, however, as in working from the crank end of an upright cylinder, it brings the instrument out of easy reach of the line from the reducing motion, and this line should be kept as short and direct as possible. The most advantageous method of connection will usually be found to be by tapping through the cylinder wall into the counter-bore, as at *A*, Fig. 33. Whether this will be at the side as in Fig. 35, or top as in Fig. 34, of the cylinder will depend upon the location of the steam chest and the direction of the cord. Usually in the larger engines with vertical cross-head the indicators are most conveniently located at the side, while in the small self-contained engines, with horizontal cross-head, the indicator is most accessible on top of the cylinder.

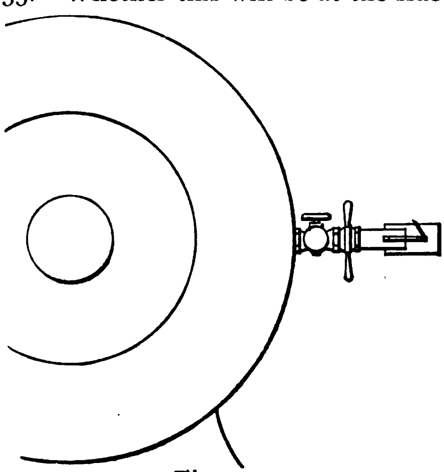


Fig. 35.

Having determined where the indicator is to be located, drill and tap the cylinder for a

half-inch pipe thread, being careful to see that the hole is not covered by the piston, but that it is in free communication with the cylinder at all points of the stroke. When the counter-bore is too close and the clearance small, access may be had by chipping a channel from the tapped hole ont into the clearance. Of course every attention should be paid to cleaning out chips and borings so that the cylinder may not be cut.

Into the hole so prepared, screw the indicator cock direct whenever possible. When the cylinder is tapped upon the side, this will bring the instrument horizontal, as in Fig. 35, but the author much prefers this arrangement to the more common one shown in Fig. 36, where a nipple and elbow are used to bring the indicator into a vertical position. The shorter and more direct the connection between the cylinder of the indicator and the engine, the more accurate will be the results, and it must be remembered that all the pipes and connections to be filled with steam represent so much added clearance to the engine, which on a small machine might amount to a considerable percentage.

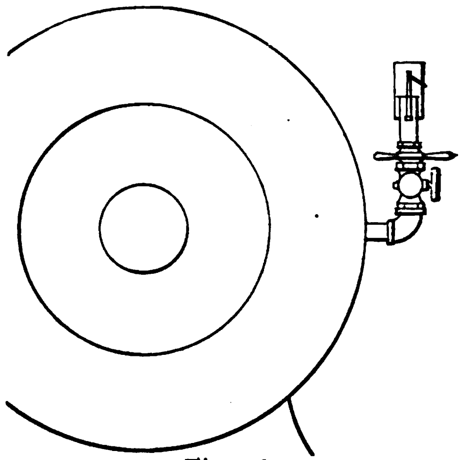


Fig. 36.

In all cases where accuracy is important, a pair of instruments should be used, one on each end of the cylinder, and diagram taken simultaneously. Where only one indicator is available it is more convenient to attach it to a three-way cock connected with both ends of the cylinder, so that it may be thrown into communication, first with one end and then the other, as at Fig. 37. This method cannot be depended upon for accuracy, however, and no important changes or deductions which could be affected by the intermediate connections should be made from the indications of an instrument attached in that way. Its convenience, however, will lead to its continued use in cases where a single instrument

is in frequent use upon the same engine; and if proper allowance is made for the distortions produced by wire drawing, condensation, and clearance, no harm will result.

A proper precaution is to take a diagram with the indicator attached directly to the cylinder, and then take another through the three-way cock, under as nearly as possible the same conditions, upon the same paper. This will enable you to make an intelligent estimate of the difference due to the different methods of connection. We have seen diagrams taken with the three-way cock which could scarcely be distinguished from those taken with the direct connection, while others have shown distortions which utterly unfitted them as indications of the

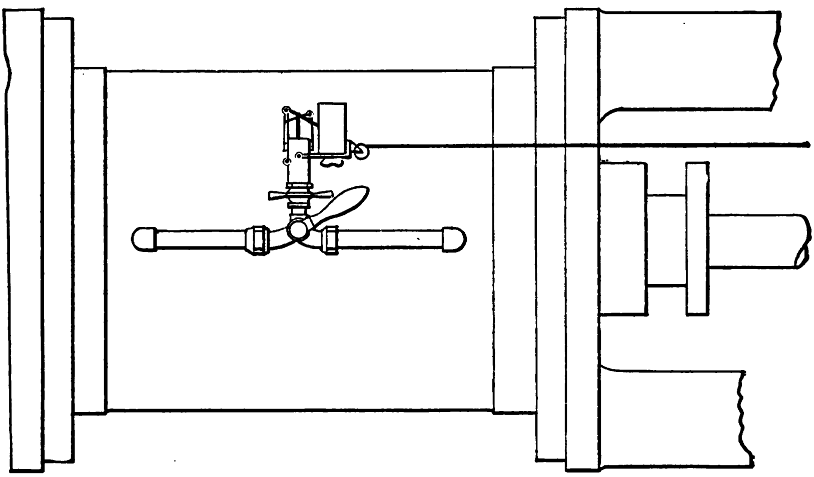


Fig. 37.

action of steam in the cylinder.* The side pipes, when used, should be well protected from radiation, and while ample in size to convey the steam to the indicator without wire-drawing, should not be any longer than necessary, on account of the increase in clearance.

The method of connection shown in Fig. 38 is to be especially avoided. Here angle valves are attached to the ends of the cylinder and connected with a side pipe, in the center of which is a T for the insertion of the indicator cock. To connect the indicator with either end of the cylinder, the angle valve at that

* See chapter on Errors in the diagram.

end is opened, the valve at the other end being closed. It is evident that in order to get any pressure to the indicator the *entire length* of the side pipe must first be filled with steam at each stroke; and for every reason that the ordinary side pipe is bad, this is twice as bad. There is also no knowing whether the valve which is supposed to be shut is tight, or whether it is entirely closed every time. Should it remain slightly open, as is frequently the case, even when a valve feels tight, some unaccountable effects may appear in the lines of the diagram taken supposedly from the other end alone.

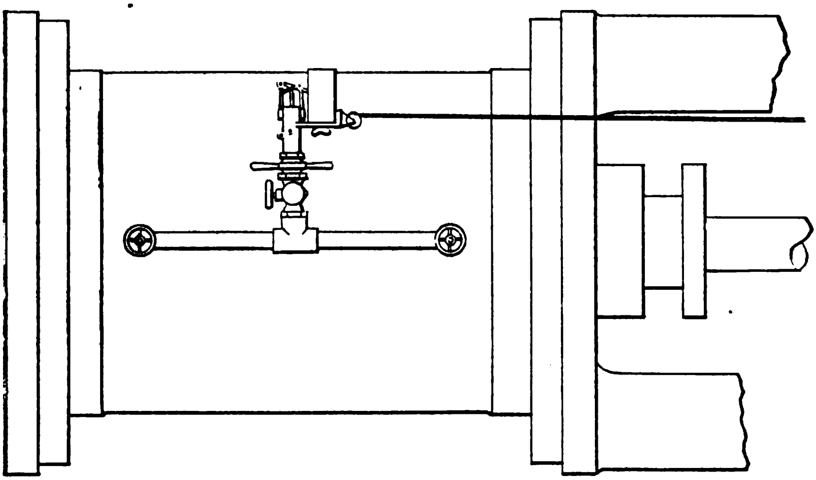


Fig. 38.

In putting up piping or connections for use with the indicator, use no red lead or other mixture, as it will be carried by the steam to the indicator cylinder and produce trouble by sticking the piston up. A few drops of oil on the thread is usually all that is required, but should a joint persist in leaking, a string of waste wound in the thread will make it tight.

Particular pains should be taken to remove from all pipes and fittings all dirt, scale, and burr which can become detached and work into the cylinder. A little piece of grit upon the indicator piston can cut some funny freaks upon the paper-barrel, as well as leave its mark upon the walls of the indicator cylinder. When the connections are all up, allow the steam to blow through them

freely some time before attaching the instrument, rapping the pipe sharply in the meantime, to remove any scale or dirt which is liable to become detached.

The cylinder having been tapped and the reducing motion arranged, we are now ready to apply the indicator to the cylinder; and here is where we begin to appreciate the fallacy of making indicators in pairs right and left, for if one is right for the side of the engine you are upon, the other is certainly wrong. You are bound to want either two right hand indicators or two left hand indicators at the same time, and when the makers recognize this and make their instruments so they can be changed from right to left, there will be fewer burnt knuckles and less profanity connected with the use of the indicator. The owner can adapt his instrument to the change by simply filing a slot in the bottom of the barrel opposite the present slot, so that the clips and pencil bar may be brought to that side of the instrument which is away from the cylinder when in use.

Do not undertake to turn the instrument backwards to bring the clips on the outside, but in putting the instrument upon the cock, let the arm which holds the barrel point in the direction which the string is to lead. It is better to take off the working parts of the instrument and leave them in the box while doing this, avoiding the risk of bending the levers and connections in handling, or catching them on the cord while rigging up. Put a little waste in the cylinder meanwhile.

The connection between the paper-barrel and the reducing motion may be made with a flexible cord, as the drum is rotated in one direction by a spring. It has already been explained that in order to secure a distribution of the pressure on the diagram corresponding to the distribution in the cylinder, it is absolutely essential that the paper-drum shall exactly correspond in its movement with the movement of the piston. To secure this, even with a correct reducing motion, it is essential that there shall be no stretch in the cord which forms the connection, through the reducing motion and cross-head, with the piston.

If the engine piston has to move an inch before the stretch is taken out of the cord sufficiently to enable it to start the drum, it is evident that the admission end of the diagram produced will not represent correctly the action of the steam with reference to the beginning of the stroke. The distortions produced will be

explained in a chapter devoted to the errors to which the diagram is liable. It is enough now to appreciate that no stretch is allowable if accurate work is to be done.

A closely braided cord, prepared especially for indicating purposes, is supplied by dealers in the instruments. It is well to hang a heavy weight upon this cord, and allow it to remain suspended some time before using, to take out any tendency to stretch which may remain in it.

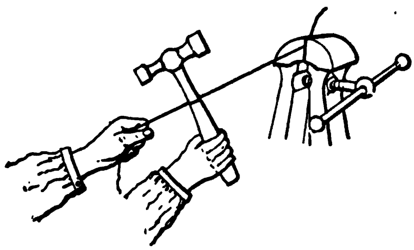


Fig. 39.

Where the distance from the indicator is considerable, as in the case of a Corliss engine, with the pantograph in the middle of the guides, the author uses, instead of a cord, annealed iron wire of about 22 gauge. This wire is subject to occasional breakage, but does not stretch, and a dime will buy enough of it to serve for many applications. It should be straightened and all the kinks taken out by being make fast at one end and wrapped about a round piece of wood, such as a screw-driver handle or hammer handle, as shown in Fig. 39, which is drawn along for the length desired. Braided picture cord wire of small size is also recommended for this purpose.

Whatever is used to lead to the reducing motion, the closely braided cord referred to will be used to run over pulleys and around the paper drum. Such a piece, terminating with a small wire hook, will be found attached to the instrument when purchased, the hook being intended to engage in a loop at the end of the cord leading to the reducing motion. If such hook is used, it should be kept as close to the instrument as practicable, as if it is some distance out it is liable to cause the line to vibrate disagreeably, especially when the speed is high. When the distance from the indicator to the reducing motion is short enough to make the use of cord advisable, the author prefers to dispense with the hook altogether, using a cord on the instrument long enough to loop over the pin in the reducing motion, and hooking on and unhooking at that point. This gives a smooth continuous line, free from loops, knots, and other incumbrances,

which will not only look better but run smoother, "stay put" better (for knots and loops are always giving and stretching more or less), and give more satisfactory results.

There are a number of other advantages which point to the reducing motion as the place for hitching and unhitching, rather than having a hook at the indicator. It is usually easier to attach the cord at this point. When the indicators are unhooked there is no attached cord being whipped about by the motion, and where a pair of instruments are used, the throwing on or off of one loop is made to start or stop the pair. There are circumstances, however, where this is impracticable, and the hook near

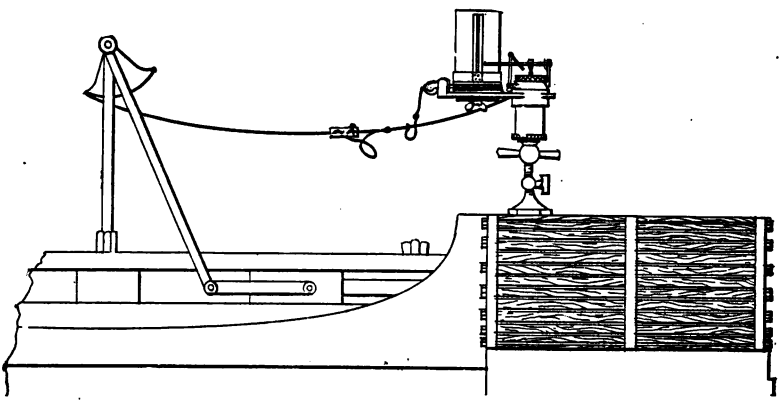


Fig. 40.

the indicator must be used. To keep the moving cord out of mischief when not attached to the indicator, it may carry the hook, the loop being made in the indicator cord, and be hooked into an elastic band attached to or near the indicator when not working the paper drum. Another method is to attach one end of the cord to the indicator, as in Fig. 40, leaving it long enough not to pull tight with the extreme motion, and looping it near the indicator for hooking on.

In any event, the end of the cord or wire which goes over the reducing motion pin should be looped, to permit the pin to turn easily within it, and not tied down closely upon the pin as by a slip-knot.

The next step is to adjust the length of the cord so that the diagram may come in the center of the card. With the indicator

in position and the engine in motion, loop the cord between your fingers and put it over the pin or hook, drawing it up enough to set the paper-barrel in motion and clear the stop. Now draw the cord carefully up until the barrel touches the stop on the outward stroke, then let it slip back through your fingers until it touches very lightly on the backward stroke. Midway between these two positions is where the point of the loop ought to be. Take back nearly half as much cord as you have let slip past, tie the loop, and the length should be pretty nearly right. Do not throw the tied loop over the pin, however, nor hook it on, until you have first held it against the pin or hook while the motion is running and made sure it is long enough. If it is hitched on too short, something is bound to give way. If, when you get to taking diagrams, it is found to be desirable to move them a little toward one end or the other of the card, this may be done by knocking the indicator around in the cock enough to take up or let out the required amount of cord. This is better than tying knots in the cord to take it up, as is frequently done.

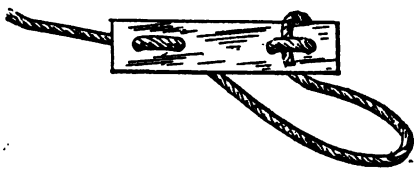


Fig. 41.

A device which may be used for adjusting the length of the loop if desired on slow speeds is shown in Fig. 41. It may be made of a small piece of sheet brass, of sufficient thickness to be stiff, in which are drilled four holes about a quarter of an inch apart. Pass the end of the cord up through the first hole, down through the second, up through the fourth, down through the third, and out over the side and under the loop, as shown. This link may be slid along upon the cord, lengthening or shortening the loop, but under the strain of the paper drum spring it will remain where placed.

Be very sure that the passage to the cylinder is free and that the piston does not even partially obstruct it at the end of the stroke. The beginning of the stroke is when the indicator makes its quickest movement, and a choking of the passage will produce apparently unaccountable results. By throwing a ray of light into the hole tapped for the indicator you can satisfy

yourself as to the directness of the passage and perhaps get a point as to evening up your clearances besides.

The tension of the drum or barrel spring should now be seen to. When the engine is making its outward stroke this drum is put into motion, and, having mass, acquires a certain amount of momentum, so that when the piston arrives at the end of its stroke and the string stops pulling, the drum continues to move by reason of its momentum until its stored energy is absorbed by the spring. If a high speed engine be run at a very moderate speed and an atmospheric line be drawn, then with the engine running at governor speed if another line be drawn just above it, there will be found to be a difference in the length of the lines of possibly ⅜ of an inch. This produces a distortion in the diagram, of course, and can be reduced by tightening up the barrel spring. For high speed engines this spring will have to be kept under considerable tension, but on slower moving machines it may be let down, and should in all cases be run only tight enough to keep the barrel well under the control of the cord.

The working parts are now to be arranged and the instrument put together. The pencil lever must be fitted with a lead. Do not use any more lead than is necessary to hold firmly in the quill or stub. Any extra weight is especially to be avoided at this point, where it has so much motion, and if allowed to stick out on the barrel side of the arm it furnishes a lever to work itself loose in the holder or to twist the pencil arm sideways in its bearings. Bring the lead to a fine round point, not sharp enough to catch in and scratch the paper. This can best be done by finishing with a very fine file or Scotch stone. Then let it stick through as little as possible, leaving a little stock for filing up the point as it wears on the side toward the paper, and break it off short at the other side.

In selecting a spring, be sure to get one stiff enough. The maximum pressure allowable with the different springs has been referred to before. If these limits are not exceeded, no harm will result to the springs or the instrument, but it may be found desirable to use stiffer springs to secure freedom from excessive vibration at high speeds. Attach the spring selected in its position, being careful to screw everything up to its place, put a few drops of cylinder oil on the piston, open the cock on

the indicator and let the steam blow once or twice through the cylinder, then put in the piston and screw the instrument together. If you get the cylinder oil from the can used about the engine room, look at the piston after the oil has spread around on it, and pick off any specks of dust or grit, which will show plainly against the bright brass. If it is a condensing engine, don't open the cock when that end is exhausting, or you may make more work for your air-pump than it can conveniently handle.

When the instrument is together, take hold of the pencil lightly and try the lever for lost motion. If it can be moved without pulling at once on the spring, take the instrument apart and take up the connections. This point should be borne in mind and looked after from time to time as the taking of cards progresses, for the connections are liable to get loose, and introduce some very curious features in the diagrams. The cards should also be watched, to see that the cord connections do not stretch so as to let the pencil bring up against the clips at the end of the diagram.

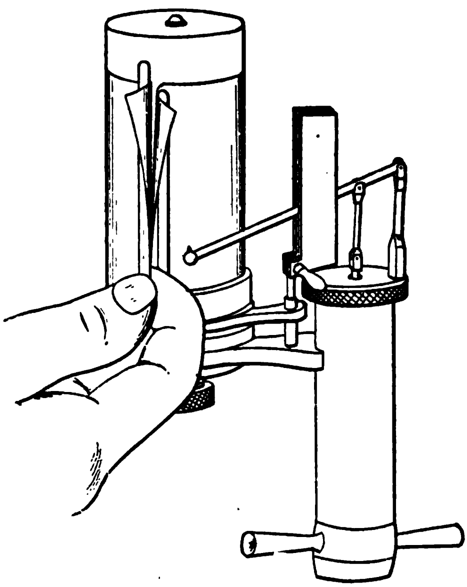


Fig. 42.

When the instrument has been put together properly, open the cock and let steam into it, setting the piston and levers in motion, and press your finger lightly on the top of the piston rod, to see if everything is working smoothly. If the least indication of gritty, scratchy action is felt, shut off the steam at once, take the instrument apart, and find the cause. If it runs smoothly, you are ready to take a diagram.

The paper used with the indicator should be a rather heavy, well calendered, smooth tough stock, something that will stand

being handled, and over which the pencil will pass without too much friction. It should be cut of such width as to reach nearly to the top of the barrel, and of a length about an inch longer than the circumference of the barrel on which it is to be used. The beginner will consider it necessary to provide himself with printed blanks, containing spaces for all sorts of observations of the engine, boiler, weather, etc.; but inasmuch as few of these observations have to be recorded on each card, and many of them, such as the dimensions of the engine, but once in a test, he will as he progresses get to using slips of plain paper, marking upon the back of each card such particulars as are needed for the purpose for which it is to be used.

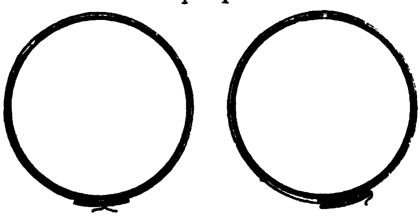


Fig. 43.

The paper is put upon the barrel by placing the lower right hand corner under the longest clip, bending it around, and allowing the ends to stick out between the clips at the top; then by taking the lower corners as they protrude between the clips between the thumb and forefinger, as shown in Fig. 42 and at the left in 43, the paper may be drawn down over the barrel as smoothly as a glove. An additional pinch near the top, and a squaring of the corners if they need it, will render the operation complete.

Another method is to put the paper under both clips, as at the right in Fig. 43. This prevents the ends from sticking out, and keeps the paper smooth. It is sometimes drawn through one clip only, as is shown in Fig. 44.

Now turn on the steam and warm up the instrument. On non-condensing engines it is well to turn the cock so that the steam will blow out into the atmosphere until it shows blue and dry. When the water has disappeared and the pencil is vibrating smoothly, the paper drum being in motion, hold the pencil lightly against the paper and allow it to trace the diagram. For ordinary purposes of exhibition, showing the valve action, distribution, etc., one revolution is sufficient to hold the pencil on. To show the governor action, variation of load, etc., the pencil will have to be held on for a number of revolutions; and

when measuring power, the pencil should be allowed to pass from ten to twenty times over, and the average diagram measured. Turn the cock off and bring the pencil again to the paper, tracing the atmospheric line. It is not good practice to trace the atmospheric line first, as the indicator and spring are not then heated, and under the same conditions as when the diagram is taken.

When through indicating, remove the spring, piston, etc., from the indicator, and allow the steam to blow through the cylinder once or twice. Unscrew the spring frcm the piston and cap, dry it thoroughly, and wipe it clean with a greasy cloth. The springs are the vital part of the instrument. Upon their integrity and accuracy the value of all your work depends. Too much pains cannot be taken to have them perfectly accurate when bought, to keep them from deteriorating by rust or otherwise, and to ascertain their condition from time to time. Wipe up and clean the levers, oiling the joints, and you will find the instrument all ready for application next time. A bottle of porpoise oil, such as is used on watches, is usually supplied with the indicator, but any fine grade of light machinery oil will answer. When the lighter parts have been attended to, the main body of the indicator will be found to be quite dry, from having had the steam blown through it, and may be cleaned like the rest and put together.

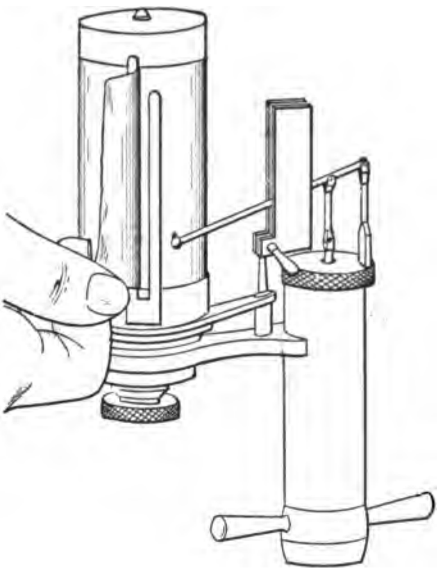


Fig. 44.

CHAPTER IV.

THE DIAGRAM.

We have learned how to correctly set up a motion, apply the indicator, and obtain a diagram. It now remains to consider what this diagram is, and what can be determined from it.

When the mathematician or statistician desires to record the results of a series of observations or experiments in such a manner that they may be at once apparent and easily comprehended, he has recourse to what is known as the graphic method. Suppose, for instance, it was desired to represent in this way the

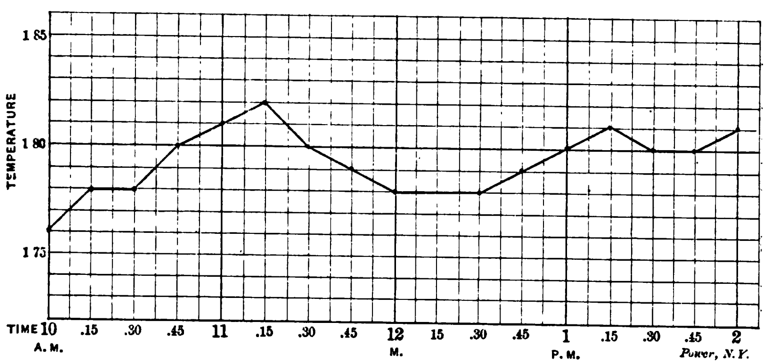


Fig. 45.

results of a series of observations of the temperature of feed-water during the test. Taking a piece of paper ruled in squares, as represented in Fig. 45 and which is known as ordinate paper, we set off the time upon one of the horizontal lines, as shown at the bottom of the figure, allowing two spaces for each fifteen minutes. Allow each of the vertical divisions to represent one degree of temperature, making the lines so figured correspond to 175, 180, and 185 degrees. Now at 10 o'clock the observation showed 176 degrees, so upon the line representing that time, and

at a height representing 176, we make a dot. Fifteen minutes later the temperature had gone up to 178 degrees, and upon the line representing 10.15 and at a height representing 178 another dot is made. Continuing in this way to represent the results of each observation, and connecting the dots by lines, we obtain a diagram showing at a glance how nearly regular the pressure was maintained through the test, to what extent it varied, and at what time variations occurred.

Let us apply this method to the variations of pressure in the cylinder of a steam engine. Suppose we have an engine with a

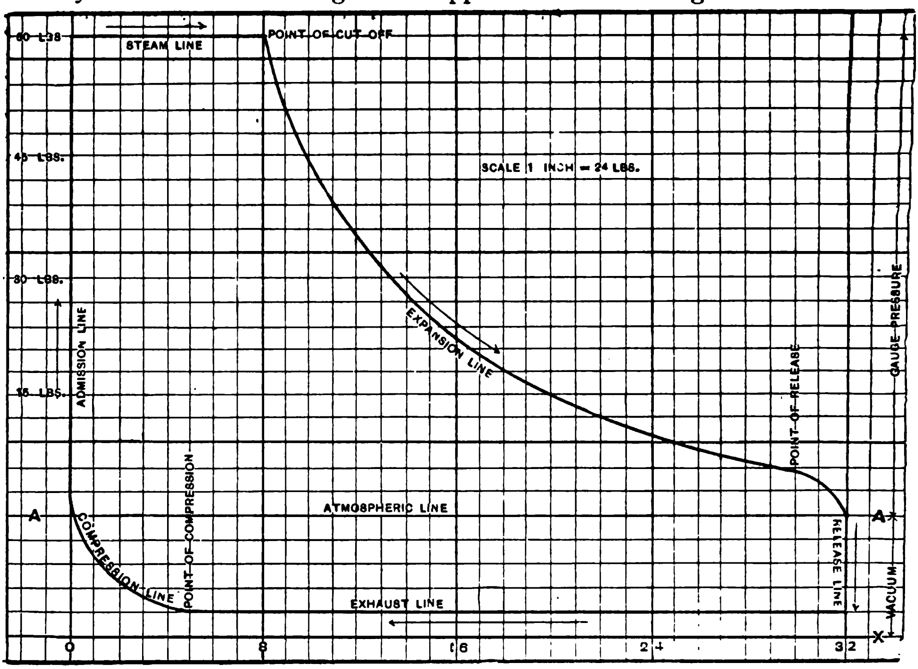


Fig. 46.

stroke of 32 inches, working with steam of 60 pounds gage pressure and a vacuum of 12 pounds, cutting off at 8 inches, with the exhaust valve opening for release when the piston is 2 inches from the end of the stroke and closing for compression when the return stroke is within 5 inches of completion.

Upon a sheet of paper ruled as in Fig. 46 draw the line *OX*, 32 spaces long, which will represent the 32 inches of the stroke, so that we can represent the successive positions of the piston or

volumes by proportional distances from *O* upon this line. We will also consider each of the spaces in a vertical direction to represent 3 pounds pressure, and starting with *OX* as the zero line can lay off to this scale the pressures corresponding to the different positions of the piston, the point *O* being the zero point of both volumes and pressures.

In the first place since the pressure of the atmosphere is 15 pounds, approximately above the absolute zero of pressure; we will lay off the line *AA*, five spaces above the zero line, to represent that pressure; and as gage pressures are reckoned from the pressure of the atmosphere as zero, we will lay off above the atmospheric line 20 spaces to indicate the 60 pounds of steam with which the engine is supplied; and as steam is allowed to enter freely for one-quarter of the stroke, we will draw the "steam line" at this height and 8 of the horizontal spaces in length. At this point the supply is cut off, and the volume of steam inclosed allowed to expand, the pressure decreasing practically in an inverse ratio to the volume; so that when the piston has arrived at the vertical line 16, and the volume has been doubled, the pressure will be halved; at the line 24, where the volume is 3 times that at the point of cut-off, the pressure will be one-third, etc., and we can calculate the pressure for each ordinate, as the vertical lines are called, and lay out the curved expansion line, as will be more fully explained when we come to consider that line particularly. At a point in this line two inches from the end of the stroke the exhaust valve opens, locating the point of release, and the pressure falls away to that of the condenser, 12 pounds below the atmospheric pressure, and 3 pounds above the zero line. Five spaces from the end of the return stroke we locate the point of compression, where the exhaust valve closes, and the steam remaining in the cylinder is compressed, as shown by the compression line, until steam is again admitted and another stroke commenced.

From the diagram thus laid out the actual action of the steam in the cylinder will vary from many causes; and an actual diagram taken from the cylinder with a steam engine indicator in which the vertical distances are determined by the pressure of the steam against a spring of known tension and the horizontal distances by a movement derived from and proportional to that of

the piston itself, will enable us, if correctly taken, to determine the actual pressure in the cylinder at each point of the stroke, and to compare these pressures, and the lines which they generate in connection with the changing volumes, with the theoretical diagram constructed as above. We are thus enabled to see how much of the available pressure is realized in the cylinder, with what degree of promptness it is admitted, and how well the pressure is maintained behind the moving piston; to observe how the valve performs its functions, how much of the vacuum is realized in the cylinder, or with what facility the spent steam is gotten rid of. We have also the data for calculating the average unbalanced pressure against the piston, and thus of determining the work performed. In fact, a properly taken diagram, with all data concerning it, is full of interest and instruction, and its study can be profitably carried to great refinement. In succeeding chapters we shall consider the separate lines of the diagram successively, show the correct form and common departures therefrom, with their causes, and lead up to calculations from the diagram, of the power developed, steam consumption, etc.

RECAPITULATION.—MOVEMENT OF THE PISTON AND THE ACTION OF STEAM IN THE CYLINDER.

We give below a tabulated summary of the entire diagram showing the formation of the various lines composing it. Reference will be had to Fig. 46.

ADMISSION LINE—During the formation of this line, steam is admitted into the clearance space, raising the pressure from the pressure of compression to steam chest pressure.

STEAM LINE—Piston is moving ahead and steam is being admitted behind it.

EXPANSION LINE—At the point of cut-off, the steam port closes and the steam behind the piston expands into a gradually increasing volume and a gradually falling pressure.

RELEASE LINE—At the point of release the exhaust port opens, releasing the pressure. The steam rushes suddenly into ths exhaust chamber, the pressure falling rapidly meanwhile.

EXHAUST LINE—By the time the piston has started on its return stroke, the pressure has reached its minimum and the

piston makes its return stroke, pushing out before it through the exhaust port the steam which has just been used in propelling it through its forward stroke from 0 to 32.

COMPRESSION LINE—At the point of compression the exhaust port closes, confining in the cylinder a small quantity of steam at a low pressure. This steam fills the clearance space and the end of the cylinder up to the face of the piston. As the piston completes its return stroke, this confined steam is compressed into a continually decreasing space, its pressure rising meanwhile, until at the lower end of the admission line the steam port again opens, admitting live steam which runs the pressure up to gage pressure.

CHAPTER V.

THE ADMISSION LINE.

The admission line shows the manner in which steam is admitted to the cylinder. Under normal conditions, admission takes place suddenly while the piston is practically standing still

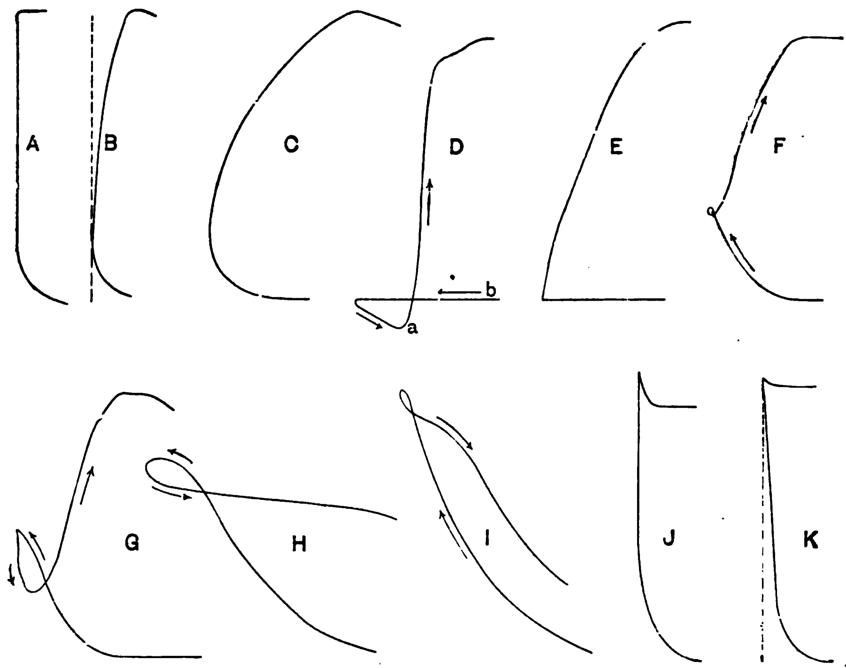


Fig. 47.

at the end of the stroke, resulting in a straight line perpendicular to the atmospheric line, into which the compression line merges, as shown at *A*, Fig. 47.

In order that the admission line may be thus erect, it is neces-

sary that the steam-valve shall be open so as to admit the full pressure before the piston commences to move away; and this involves the question of lead, or the amount of opening which the valve has when the engine is on the center, and which, for many reasons, it is desirable to keep as small as possible and yet allow the admission line to be perpendicular. As the steam-valve is allowed to become late in opening, and the piston gets into motion before the steam is admitted, the admission line commences to curve inward, as at *B* and *C*, the leaning tendency increasing as the line progresses and the motion of the piston becomes faster. At *D* is shown a peculiar admission line on a diagram taken by the author from a slide-valve engine, the eccentric of which had slipped so as to make the whole valve-motion late. The exhaust closure being late as well as the steam opening, the compression was entirely cut out, and the back pressure line *b* continued straight up the end of the stroke. When the piston commenced its return stroke the steam-valve had not opened. The exhaust-valve had by that time closed, the space between the cylinder head and the retreating piston was entirely shut in, and as the piston moved away a vacuum was created, running the pressure down towards *a*, as is shown by the arrow. At *a* the steam was admitted suddenly and the admission line ran up, leaving the loop on the heel of the diagram as shown.

The admission line may lean in, however, from another cause than that of the steam-valves being late, as the author found in procuring the diagram whose admission line is reproduced at *E*. The natural inference from the appearance of the diagram would be that the engine was late all around, but the fact is that the steam-valve has plenty of lead and opens before the return stroke is completed; but the exhaust-valve is so late that it not only does not close for compression, but does not close until the piston has got well started on the forward stroke, so that the steam is blowing right through into the exhaust and cannot keep the pressure up. As the exhaust closes, however, the pressure is increased, but the piston is moving away so rapidly that the line never becomes erect.

The amount of compression has a great deal to do with the appearance of the admission line. The effect shown at *F* is a

very common one, produced by the pressure running up by compression to the point and falling away on account of late admission as the piston starts back before the steam valve opens, forming the loop. A more aggravated case of the same action is shown at *G*, which represents the condition in which an old-fashioned, upright Corliss engine ran for a number of years. This loop assumes all sorts of forms, according to the relations of the compression and admission, and the proportions of the openings and the piston speed; and may even be formed when the steam-valve opens promptly, by excessive compression, as frequently seen on diagrams from the ordinary type of single valve, high speed engines with shaft governors, where the compression is increased as the load diminishes, resulting in admission lines like those shown at *H* and *I*. In the first of these the pressure is so low that the compression line extends above it, and when the steam-valve opens, there is an escape of steam from the cylinder and the pressure is lowered to that at which the steam will flow from the chest. The appearance at *I* is produced when the engine is lightly loaded, so that the compression is very considerable.

A sharp point at the top of the admission line is usually an indication of too much lead, and it will be found to result in smoother running if the corner is just given an indication of rounding, as at *A*. The projection is due to the fling of the moving parts carrying the pencil above the point due to the pressure.

Just as a tardy action of the steam-valve results in producing an inward leaning of the admission line, so a too early opening of that valve will result in the production of a line which leans outward, as shown at *K*. This is to be avoided, as it puts an injurious strain on all the working parts of the engine, pushing with all the force of the steam pressure multiplied by the piston area upon the crank as it is coming up over the center, and crowding the shaft hard into the main bearing to no purpose. It simply sets the steam pressure to work against the desired movement of the engine, and robs the diagram of the effective area between the admission line and the perpendicular dotted line *K*, which indicates the position the admission line should really occupy. Any engine which is in line and properly adjusted in

the connections, should run at the speed for which it is designed better with enough lead to bring the admission line upright, than it does with more, and if the upright is to be departed from at all, it had better be in the direction of making the valve late than in that of giving the engine steam before it is ready for it.

CHAPTER VI.

THE STEAM LINE.

From the steam line of the indicator diagram we are able to determine what percentage of the boiler pressure is realized in the cylinder and how well this pressure is maintained up to the point of cut-off. Steam or any other fluid will not flow without a difference of pressure between the vessel from which it flows and that into which it is delivered, and this difference in pressure must be sufficient to overcome the frictional resistance of the connecting pipes and passages. It is absolutely impossible, therefore, to maintain in the cylinder the same pressure that is carried in the boiler, although with short connections, ample passages, and low piston speeds a very large percentage can be realized.

In a really good diagram the steam line will appear about at *A*, Fig. 48, approaching, in its height above the atmospheric line, the distance indicated by the boiler pressure laid off to the same scale as that of the spring with which the diagram is taken, as shown by the dotted line, and remaining horizontal, or very nearly so, up to the point of cut-off. When the connecting pipe and passages are small for the piston speed and diameter, the linear velocity of the flow becomes so great that a greater difference in pressure is necessary to overcome the increased resistance, and the steam line falls away, as at *B*, sufficiently to keep up the difference necessary for such a rate of flow, as at *a* and *b*, the difference at *a* being sufficient to maintain the lesser velocity at the beginning of the stroke, while the greater difference in pressure at *b* is necessary when the piston has gained the greater speed due to that position in the stroke.

Such a falling away may be due either to faulty design or setting of the ports and valve of the engine itself, in which case the loss of pressure will occur chiefly between the steam chest

and the cylinder; or to a long, tortuous, or insufficient connection between the engine and boiler, in which case the loss of pressure would occur between the boiler and the steam chest.* To which of these causes the loss is mainly due, and how much of it is due to each, may be determined by applying the indicator to the steam chest, taking the motion from the cross-head just the same as when the indicator is upon the cylinder. Such a diagram should be taken by transferring the indicator from the cylinder

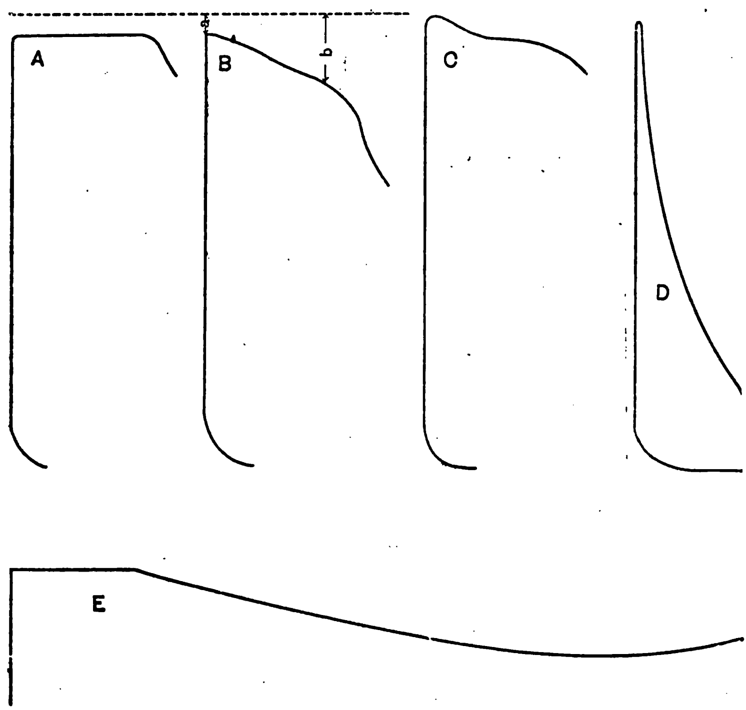


Fig. 48.

to the steam chest without disturbing the paper on which the cylinder diagram has been taken, and maintaining the boiler pressure, load and speed constant, in order to best show the relations of the diagrams. A still better way, when plenty of indicators are available, is to have an instrument on both the chest and cylinder, take simultaneous diagrams, to the same

* See Chapter XIII on Errors of the Diagram.

scale, and transfer them to one card, by making the atmospheric lines identical. This may be handily done by cutting the card from the cylinder close to the steam line at the top, and reducing its length so as only to include the diagram. Then extend the atmospheric line to the ends of the card, extend the atmospheric line on the steam chest card, and place the two cards so that the atmospheric lines will coincide as in Fig. 49, one diagram being directly beneath the other. Being made from the same reducing motion, their lengths should be the same.

The diagram shown above the ordinary cylinder diagram in Fig. 49 is a conventional steam chest diagram. At *a* the valve opens to let steam into the cylinder, and the out-rush of steam reduces the pressure in the steam chest until there is the

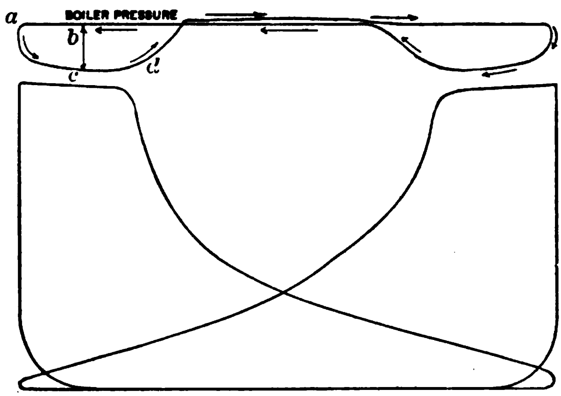


Fig. 49.

difference between the boiler pressure and the pressure in the chest indicated by the space *b c*, between the line of boiler pressure (which should be drawn in on the diagram at a height measured from the atmospheric line by the same scale with which the diagrams were taken) and the lower line of the chest diagram.

Understand, the vertical distance between the line of boiler pressure and the lower line of the chest diagram, represents the loss of pressure between the boiler and the steam chest at that point. The space between the lower line of the steam chest diagram and the steam line of the cylinder diagram at any point in the stroke is a measure of the loss of pressure between the steam chest and the cylinder. The greater the distance from the

boiler, the smaller the pipe, and the greater the number of turns, the greater the loss of pressure between the steam chest and the boiler, and the greater the area of the steam chest diagram. The smaller, longer, and more crooked the ports, the greater the reduction between the steam chest and cylinder and the greater the lost area between the diagrams. Following out the outline of the steam chest diagram, the pressure continues to fall along the line *a c d* as the piston moves faster and faster until the cut-off valve closes and the draft of steam from the chest ceases, when the pressure in the chest commences to recover and runs well or quite up to boiler pressure as the flow of steam is stopped. It may even run above the boiler pressure on account of the momentum of the moving column of steam in the connecting pipes. A similar action upon the other end completes the diagram. It will be seen that in this way the cause of any excessive loss of pressure can be located exactly and the relative importance of changes in the engine or piping determined.

In order to prevent an undue fall of pressure, and wire drawing of the steam, the passages leading to the cylinder should be so proportioned that at no point the linear velocity of flow shall exceed 6,000 feet per minute. This can be done by making the passages bear the same proportion to the cross-sectional area of the cylinder that the piston speed does to 6,000; i.e., take for the smallest cross-sectional area of the steam pipe or passages such a fraction of the cross-sectional area of the cylinder as is indicated by writing the piston speed in feet per minute as a numerator over 6,000 as a denominator.

For a piston speed of 600 feet per minute, for instance, the smallest cross section of the pipe or port should not have an area less than $\frac{600}{6000}$ or one tenth of the cross-sectional area of the cylinder.

On engines with large steam chest capacity the appearance at *C*, Fig. 48, is often met, the large volume of steam already at hand sufficing to keep the pressure up at the commencement of the stroke, but when the piston movement becomes more rapid and the draft from the boiler begins in earnest, a greater difference in pressure is required to maintain the flow, and the line drops to the lower level, as shown.

If there is any tendency to fall away on the part of the steam line, it will, under equal conditions, manifest itself most decidedly on the head end of the cylinder, as the piston movement is faster on that end, owing to the angularity of the connecting-rod.

The downward tendency of the steam line increases with its length, for as the stroke progresses, the velocity of the piston movement becomes greater and the rate of flow accelerated. It is therefore very rarely that we find a long steam line on a cut-off engine, which does not commence to fall away seriously from the initial pressure, although it may hold up nicely during the earlier portion of the stroke.

A decided example of this action is seen in diagrams from cut-off engines when cut-off does nót take place. Such a diagram is shown at *E*, Fig. 48, and it will be seen that although the steam line is well maintained at the commencement of the stroke, the steam follows the piston with more difficulty during

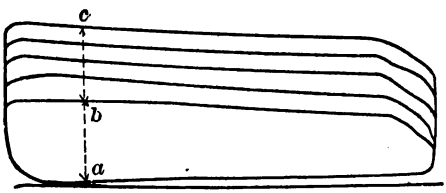


Fig. 50.

the rapid movement in the middle of the cylinder and the pressure falls away, recovering somewhat as the movement grows slower on approaching the other end. The same effect is observable at times upon diagrams from throttle-governed engines; but as the steam lines of such diagrams depend upon the vagaries of a governor situated between the cylinder and the source of steam supply, little interest attaches to their study as denoting the action of the steam.

In throttle-governed engines the area of the diagram, which is the measure of the amount of work performed, is varied in accordance with the demands of the load by increasing the vertical distance between the steam line and the line of counter pressure, as from *a* to *b* (Fig. 50) for a light load and from *a* to *c* for a heavy load, while in the automatic cut-off engine the same object is effected by varying the length of the steam line by

cutting off the steam earlier or later in the stroke, as from *A* to *B* (Fig. 51) for a light load and from *A* to *C* for a heavy load.

Diagrams are sometimes met with which have no steam line, the load being so light that the expansion of the steam in the clearance is sufficient to keep the engine in motion. In this case the expansion line meets the admission line at a point, as at *D*, Fig. 48.

The shape of the steam line is often modified by the admission,

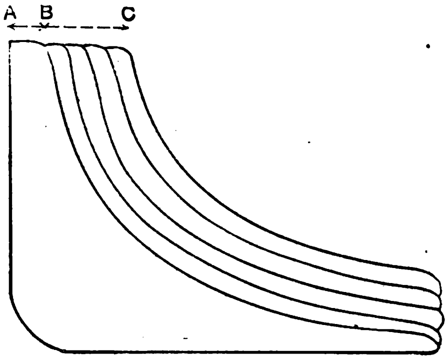


Fig. 51.

and it will be realized from the remarks about the admission line in the last chapter that it is difficult to say when the one leaves off and the other begins, under frequently occuring conditions.

CHAPTER VII.

THE EXPANSION LINE.

In all engines in which any pretension is made to economy, steam is used expansively, the supply being cut off at some point in the stroke, determined either automatically by the governor or positively by the valve. By this means the piston is urged not only while there is a direct draft of steam from the boiler, but by the expansive force of the steam in the cylinder after this draft has ceased.

Referring to Fig. 52, let $o\ x$ represent the stroke of an engine, and oA the pressure of steam in the cylinder at the commencement of the stroke; then, since the power is the pressure multiplied by the space through which it is exerted, we should have for the power of a cylinder in which the initial pressure is continued to the end of the stroke a value proportional to the area of the rectangle $ABxo$, and the cylinder would require to be completely filled with steam from the boiler at each stroke. If instead of allowing the steam to follow full stroke we cut off the supply at mid-stroke, as indicated at C, we shall have behind the piston at this point a half-cylinder full of steam at the initial pressure, which, as the piston moves onward, will be expanded, allowing its pressure to fall along the curved line CD. The power generated will now be proportional to the area $ACDxEo$, less by the area BCD than it was before; but the amount of

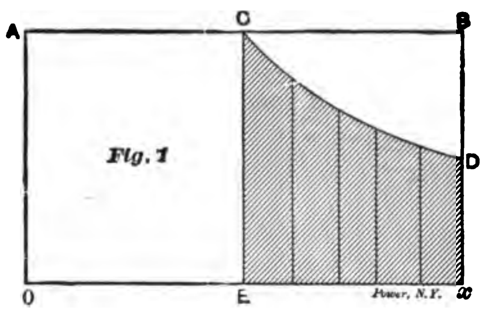


Fig. 52.

steam called for from the boiler has been only one-half as much as when the engine followed full stroke, and the power represented by the shaded area $CDxE$ has been gained at no expense for extra steam.

Steam in expanding in an engine cylinder under the conditions of ordinary practice varies in pressure so nearly in an inverse ratio to its volume that we can use this law in laying out the approximate path that the curve CD, Fig. 52, will take.

Supposing an engine with a 48-inch stroke to cut off at 8 inches or one-sixth of the stroke, with steam of an absolute pressure of 90 pounds, 75 pounds by the gage. Representing the stroke of this engine by the base line of the diagram Fig. 53, we should have,

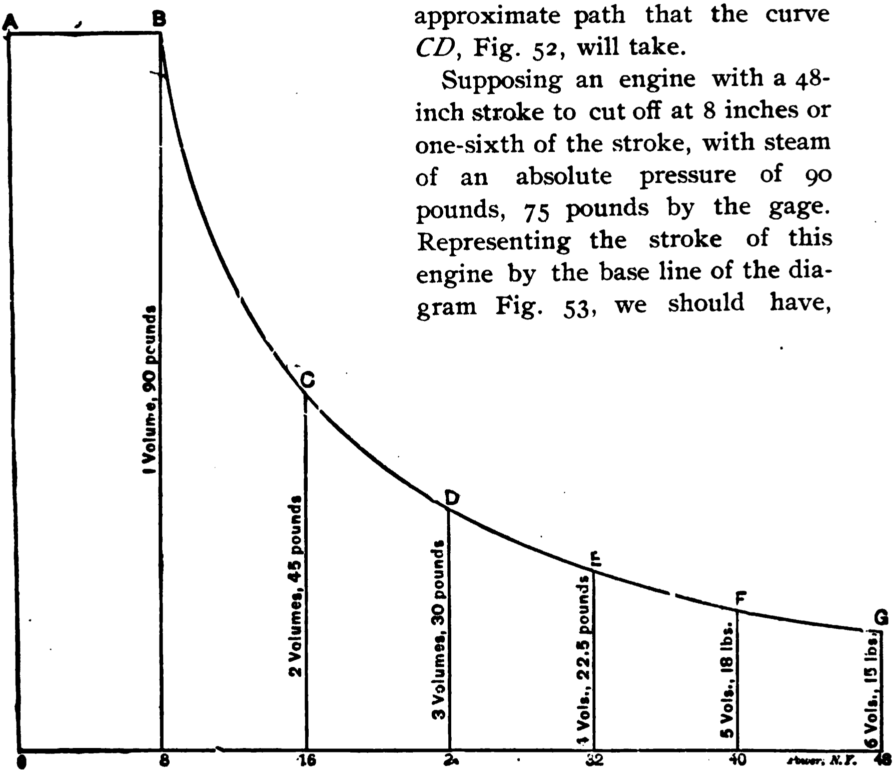


Fig. 53.

when the piston had completed the eighth inch of its stroke, one-sixth of the cylinder full of steam at 90 pounds pressure, represented by the area $oAB8$. The supply is now cut off, and when the piston has arrived at the 16-inch point the steam will have expanded to double its volume at cut-off, and its pressure will be reduced to one-half or 45 pounds, represented by the height of the point C. When the piston had proceeded

another 8 inches, or to the 24-inch mark, its volume would have been trebled and the initial pressure divided by three, giving a pressure at this point of 30 pounds, represented by the length of the line *24D*, which is one-third of the line *8B*, representing the pressure of the initial volume.

In the same way we would find one-fourth the pressure when the steam had been expanded to four times the initial volume at *E*, one-fifth the pressure when the volume had attained five times the original at *F*, and one-sixth the pressure at *G*, where the volume is six times what it was at the point of cut-off. In this way the pressures at various points in the stroke may be calculated and set off upon ordinates representing by their position upon the horizontal line the corresponding point in the stroke, and a curve drawn through these points will be the theoretical expansion curve.

As a simple rule for finding the pressure at any point in the stroke:

Multiply the absolute pressure at the point of cut-off by the fraction made by writing the number of inches of the stroke completed at cut-off as a numerator over the number of inches completed at the given point as a denominator.

For example, to determine the pressures at *C*, *D*, *E*, *F*, *G*, in the above described diagram we have:

At C the pressure $= \frac{8}{16} \times 90 = 45$ pounds
D " $= \frac{8}{24} \times 90 = 30$ "
E " $= \frac{8}{32} \times 90 = 22.5$ "
F " $= \frac{8}{40} \times 90 = 18$ "
G " $= \frac{8}{48} \times 90 = 15$ "

Notice also that *the product of the volume and pressure is constant.* At *B* we have one volume and 90 pounds and

		Volume.		Pressure.		Product.
At *B*	=	1	×	90	=	90
" *C*		2	×	45	=	90
" *D*		3	×	30	=	90
" *E*		4	×	22.5	=	90
" *F*		5	×	18	=	90
" *G*		6	×	15	=	90

The curve may be laid out geometrically in a number of ways,

which we will proceed to show. In applying it to an indicator diagram, however, the fact must be taken into account that beside the volume of steam represented by the piston displacement up to the point of cut-off there is the steam in the clearance spaces, which will share in the expansion, and the initial volume must be made to include this steam. We will apply the curve to the diagram in Fig. 54 by one of the simplest methods. This diagram is 4 inches in length, and we will assume a clearance of 2½ per cent. Two and a half per cent of 4 inches

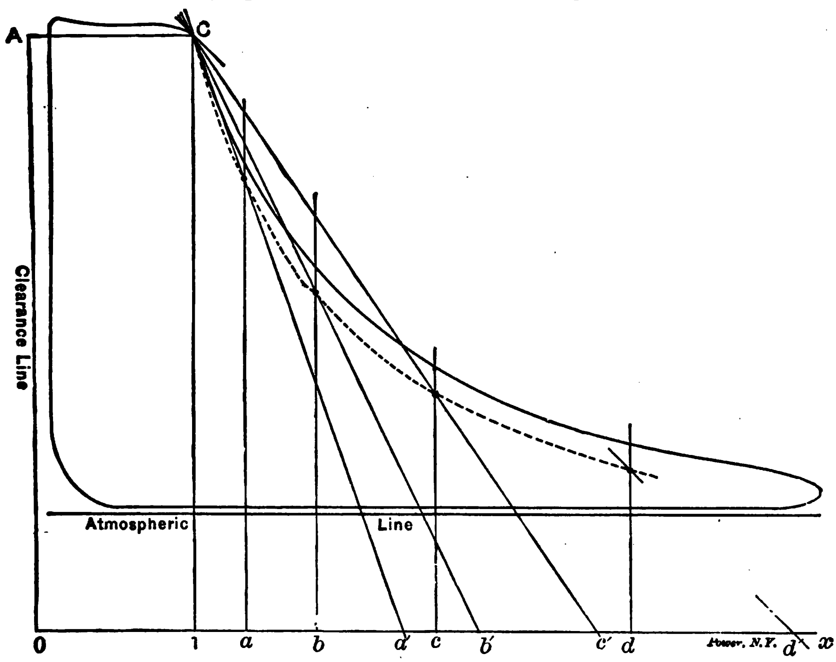


Fig. 54.

is one-tenth of an inch, by the addition of which we will increase the length of the diagram at the admission end by drawing in the clearance line *Ao* one tenth of an inch from the extreme end of the diagram. Draw the line of absolute pressure 14.7 pounds below the atmospheric line. With ordinary high scales 15 pounds is sufficiently accurate. Now at the point of cut-off *C* there will be in the cylinder a volume of steam proportional to the area *AC1 o* of a pressure proportional to the line *1C*. At

right angles to the line of absolute zero, *ox*, erect perpendiculars at points where it is desired to locate the curve. As the curve changes more rapidly just after cut-off, it is advisable to put in these perpendiculars more closely in the earlier portion of the stroke, as shown, and this is especially true of diagrams with large ratios of expansion, i. e., early points of cut-off. Now take in the dividers the width of the space representing the initial volume, i. e., the length of the line *AC* or *o1*, and from the base of the first ordinate *a* measure off an equivalent distance *aa'*=*AC*, upon the zero line. A line connecting the point of cut-

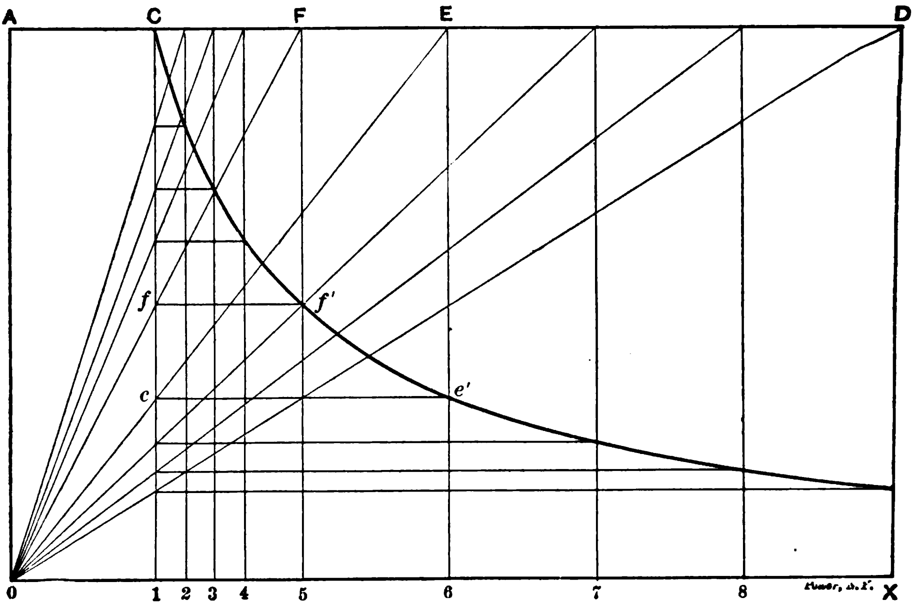


Fig. 55.

off *C* with *a'* will cross the vertical ordinate *a* at the point through which the curve must pass. From the base of the second ordinate *b* set off the same distance *bb'*=*A C*, and a line joining the point of cut-off and *b'* will cut the ordinate *b* at the point through which the curve should pass at that point of the stroke. Proceeding in this manner with *c* and *c'*, *d* and *d'*, and as many other ordinates as are essential, the points through which the curve will pass may be located and the curve traced in as indicated by the dotted line. In practice it is not necessary to

draw the lines from the point of cut-off, but simply to mark the point at which the straight edge crosses the ordinate, as shown upon the ordinate *d*. By spacing the ordinates *a b c*, etc., the same distance from each other that the point of cut-off is from the clearance line, i. e., making the distance between the ordinates equal to *AC* or *o1*, the base of one line may be used as the point from which to rule to the point of cut-off to locate the curve on the preceding ordinate, but this method does not with ordinary diagrams give a sufficient number of ordinates to locate the curve accurately in the earlier portion of the stroke.

Another method frequently used for laying out the theoretical curve is shown in Fig. 55. Allow *OX*, as in the previous examples, to represent the line of absolute zero, the line *AC*, by its distance from the zero line, the initial pressure, and by its length the volume of steam up to the point of cut-off, including that in the clearance, determined as previously shown. Erect any number of perpendicular ordinates, as 1, 2, 3, 4, 5, 6, 7, 8, at points where it is desired to locate the position of the curve. Continue the line *AC* for the full

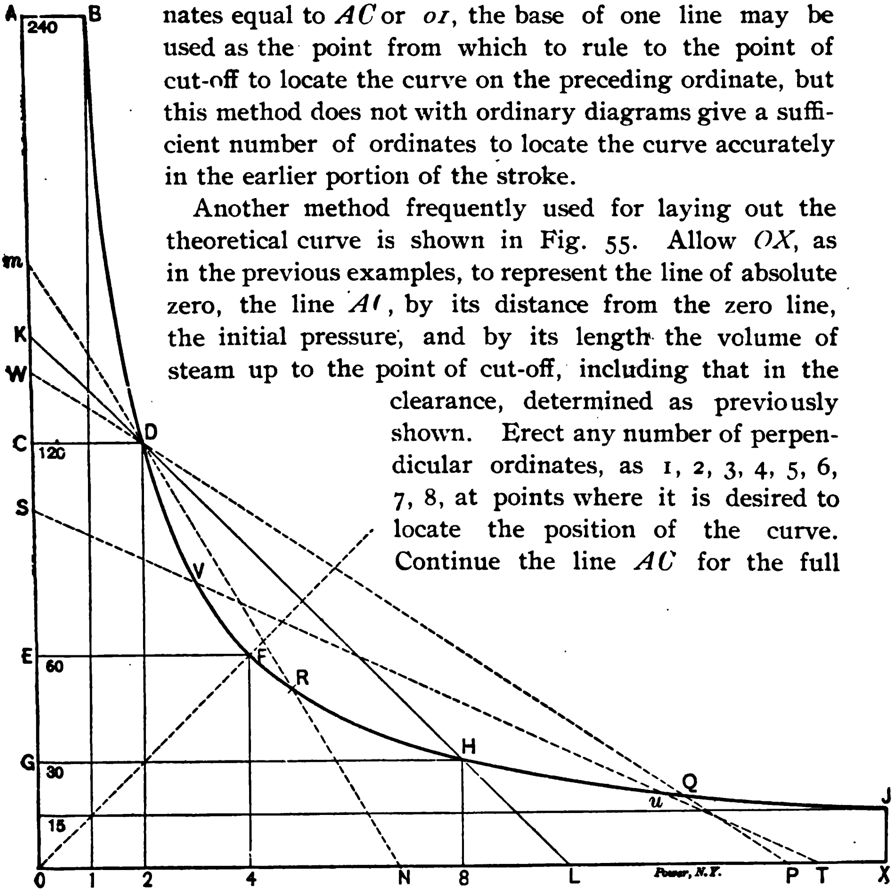


Fig. 56.

length of the diagram *AD*. The point through which the curve would pass on any ordinate, as *6* for example, is found by connecting its top *E*, as determined by the line *AD*, with the point o. The line *Eo* will cross the line *1C* at the point *e*, which indicates the height at which the curve would pass on the line *6E*,

and may be transferred to that line by drawing the horizontal *ee'*. In the same way the point *f'* is located upon the ordinate *5F*, and at as many other positions as are necessary to determine the course of the curve with the necessary accuracy.

The curve which we have been describing, and which corresponds with a constant product for pressures and volumes, is a rectangular hyperbola; rectangular because the asymptotes, as the lines *oA* and *oX* are called, are at right angles. Let the rectangle *oAB1*, Fig. 56, represent by its height the pressure and by its width the volume of an amount of steam. The area of an rectangle representing this amount of steam at any other volume (the pressure changing accordingly) will be the same as the area of *oAB1*, for the area is the product of height and width, which represent respectively the pressure and volume, and with hyperbolic expansion the product of the pressure and volume is constant, as shown on page 63. With the volume doubled, therefore, the rectangle representing the new condition would be *oCD2*, one-half the height and twice the width, and at 4 volumes the rectangle becomes a square, the lines representing the pressure and volume being of equal length. After this point the lines representing volumes become longer than those representing pressure, but we shall have simply a repetition of the rectangles for the earlier volumes with their length horizontal instead of vertical. The rectangle of *oGH8*, representing 8 volumes and 2 units of pressure, is the same as the rectangle *oCD2*, representing 2 volumes and 8 units of pressure. Thus it will be seen that the curve is the same on both sides of the diagonal *oF*, which is called the axis, and that the portion of the curve which lies between *F* and *J* is precisely similar to that which lies between *B* and *F*.

It is a property of this curve that a line drawn across so as to intersect it in two places, as *KL*, *mN*, *WP*, will cut the curve at equal distances from the asymptotes at both ends. It is easily seen that the point *D* on the curve is the same distance from *K* that *H* is from *L*. As the top of the line is carried downward from *D* as to *W*, the distance is decreased as to *WD*, but the curvature is such as to make the distance *QP* upon the other end precisely equal. So also the increased length in the position m*D* is met by a similar increase in the distance *RN* at the other end

of the line. This is true whatever point is chosen upon the curve or whatever inclination is given to the line, so long as it cuts the curve in two places and both asymptotes. For instance, on the line ST placed at random, the distances SV and Tu are equal.

This property is made use of in several constructions used upon indicator diagrams, one of which is laying out the curve as

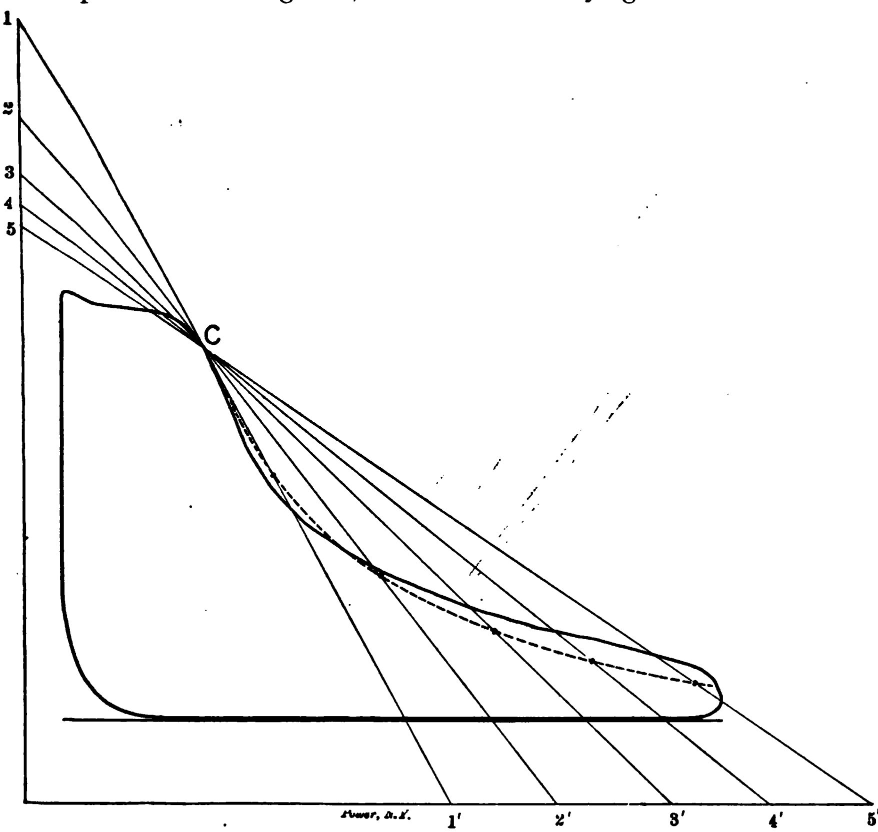


Fig. 57.

shown in Fig. 57. Through any point upon the expansion line, as C, draw straight lines to the line bounding the clearance in one direction and to the line of absolute vacuum in the other. Upon the line 1 1′ set off a distance from 1′, equal to 1C. Upon the line 2 2′, set off a distance from 2′ equal to 2C, and continue

the process upon the other lines as shown. The theoretical curve passes through the points just found. In practice it is unnecessary to draw lines, distances being laid off by means of the dividers against the edge of the ruler. This principle is also used to determine at what point cut-off should occur, assuming initial pressure to be uniformly maintained, in order that the expansion line may pass through point A. Drop a perpendicular line from A, Fig. 58, to the line of zero pressure, and connect the point B of its intersection with the point P upon the line of zero volumes, indicating by its height the given pressure. A line pb, parellel to PB and passing through the given point A, will cut

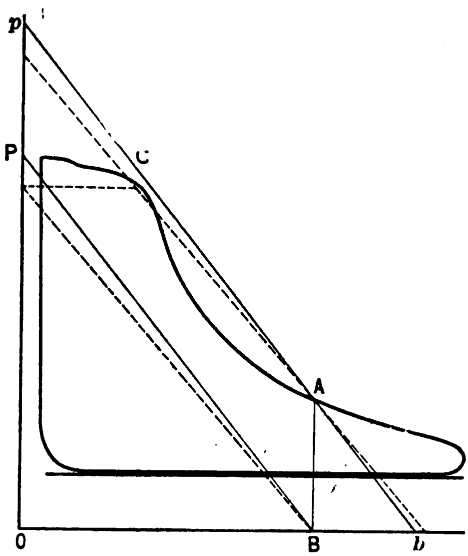


Fig. 58.

the line PC at the required point at which expansion should commence in order that the curve may pass through A. For under these conditions the triangle PpC is the same as the triangle ABb, and upon the line bp the points A and C are equidistant from the asymptotes. The point of cut-off for any other initial pressure may be determined in the same way by varying the position of the point P, as indicated by the dotted lines.

Another construction sometimes used upon the expansion line of an indicator diagram is shown in Fig. 59. This is for the

purpose of finding the position of the line OA, bounding the clearance space. From any two points, as BC, upon the established portion of the curve, draw lines as BD and CE, parallel to the atmospheric line, also the perpendicular lines BE and CD, forming a rectangle. At a distance below the atmospheric line corresponding to 14.7 pounds on the scale of the diagram draw the line of absolute zero of pressure OX. The diagonal DE of the rectangle $BDCE$ will, if continued, cut the line of zero pressure at the point O of zero volume, from which point the perpendicular line OA, the position of which we are seeking, may

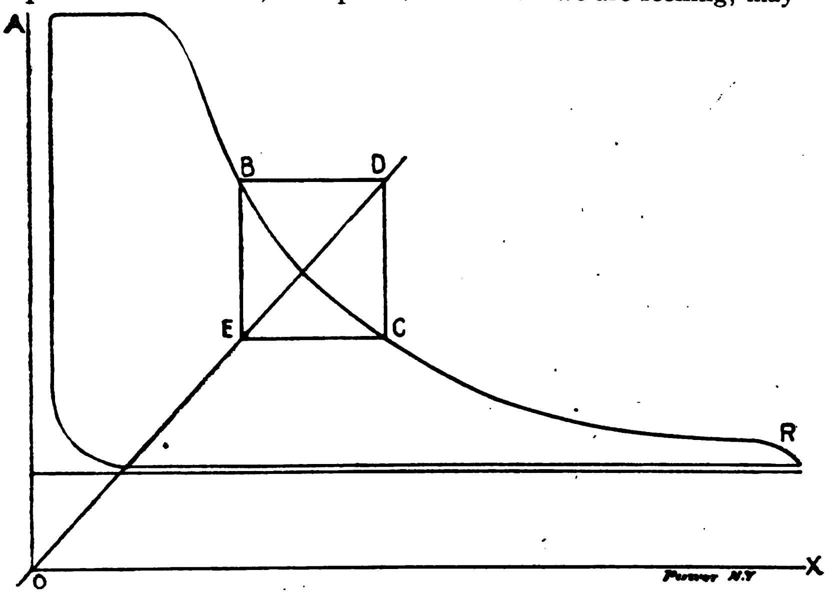


Fig. 59.

be erected. The theoretical curve is of value in showing us what, under given conditions of pressure and expansion, we may expect a diagram to be, and serving as a basis of comparison for the actual diagram. It is not precise, however, and too much stress should not be placed upon its indications unless very marked.

The law upon which the above processes are founded, that the pressure varies inversely as the volumes, is true for a perfect gas, but only approximately so for steam. The steam of practice is, moreover, heavily charged with water after its admission to the

cooler surface of a cylinder which has just been exposed to the temperature of the exhaust; and this water, by its re-evaporation during the later stages of expansion, may make a consider-

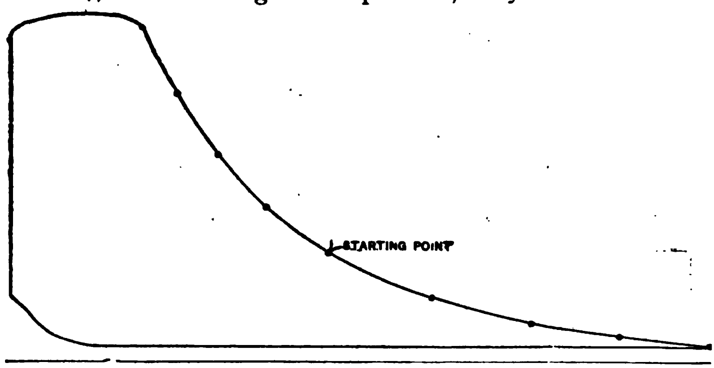


Fig. 60.

able difference in the course of the actual expansion line, especially where expansion is carried to a considerable extent. The general effect of condensation and re-evaporation is to carry the curve below the theoretical in the earlier portion of the expansion line and above the theoretical in the later portion. This may be modified by leakage into or from the cylinder. The existence of such leakage is better capable of positive demonstration, however,

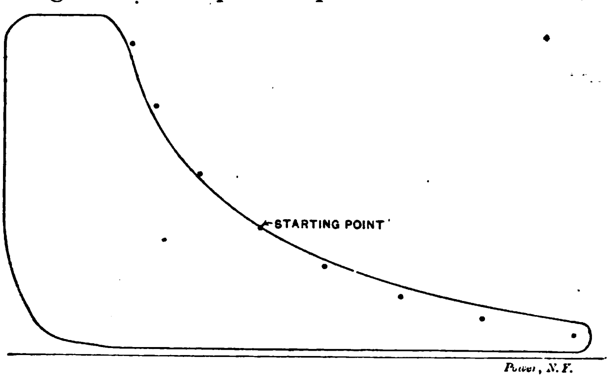


Fig. 61.

by trying the valves and piston under steam.

As an instance of this may be shown two diagrams taken by F. Ruel Baldwin, M. E., from an engine the exhaust valves of which leaked very badly. The first of these, Fig. 60, was taken

while the valves were in their leaky condition, but the expansion curve fits the line of the diagram very nicely. Fig. 61 was taken after the valve had been made tight, but there is a considerable difference between the theoretical and the actual lines.

It has been suggested that the quality of the steam will have a marked effect upon the expansion line, for if it carries a large amount of water at the temperature of the entering steam, such water is expected to evaporate when the pressure falls below its boiling point. It requires a large amount of heat, however, to evaporate water, even after it has reached the boiling point, and even the amount condensed initially from dry steam will not be evaporated during the period of expansion, so that moisture in the steam would have little if any effect upon this line from this cause.

The accompanying chart will be found convenient in comparing the expansion lines of actual diagrams with the theoretical curve. Draw upon the diagram the line of absolute zero 14.7 pounds below the atmospheric line and the clearance line locating its position by calculation as in Fig. 54 if the percentage of clearance is known, or by construction as in Fig. 59. Place the diagram beneath the transparent chart with the zero line under *O X* and the clearance line under *O A* and the theoretical curve may be studied direct or transferred to the diagram by pricking through the chart.

CHAPTER VIII.

THE POINT OF RELEASE.

When it is possible of attainment we like to see the release end of a diagram given the appearance shown at *A* in Fig. 62, the

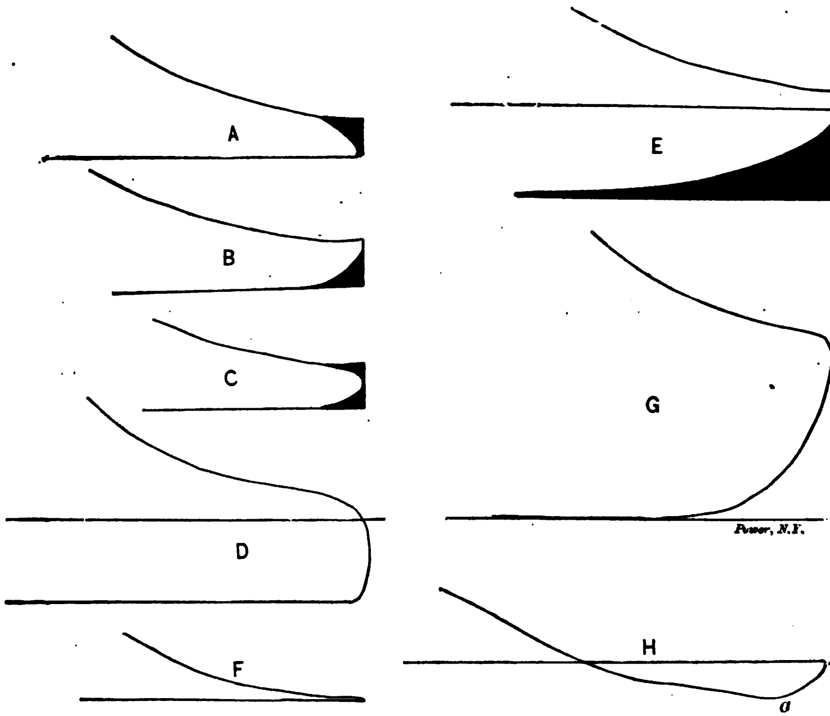


Fig. 62.

release occurring early enough to allow the pressure to fall nearly or quite to the line of counter pressure by the time the end of the stroke is reached. If the release is delayed until the end of the stroke the appearance will be more like that indicated at *B*. If the pressure could be carried to the end of the stroke and im-

mediately reduced to the line of counter pressure, as indicated by the outside edge of the black space, it would be advisable to retain the full area; but since some area must be lost here in expelling the exhaust, it is better that it should be above the diagram as at A than below as at *B*. When the piston is approaching the end of its stroke, it has come to be a question of stopping it and sending it in the other direction. To do this smoothly we apply compression on the other side of the piston, and obviously there is no object in keeping up the forward pressure, as at *B*, unless we can add to the effective area of the diagram (which represents the useful work done by the steam) by doing so. It is therefore better to let the pressure fall off, as at *A*, assisting, instead of opposing, the compression in bringing the moving parts quietly to rest, and by this early release removing the back pressure represented by the black portion at *B*, so that the piston encounters less resistance in starting upon its backward stroke when it is an object to get it into motion. In this way we sacrifice nothing in the area of the diagram, and get a better distribution of the pressures with reference to the practical work of the engine. The difficulty of attaining the result on most engines is that where the lap is removed from a valve to cause it to open early and give an early release, this very lack of lap retards the closure and does not give sufficient compression. On the Corliss valve this may be corrected by setting the eccentric ahead, making both release and compression earlier, but disadvantages attend upon too great an angular advance of the eccentric, in the way of shortening the range of cut-off, and the advantages of the valve motion in quick movement at admission, so that it is often necessary to divide the difference and compromise upon a point like that shown at *C*. The benefit of an early release is very apparent when a condenser is used, for with an early release and a prompt realization of the vacuum, as at *D*, the largest possible percentage of the load is thrown upon the condenser; while a tardy release and a dragging action of the steam in leaving the cylinder results in the loss of a large area in the vacuum portion of the diagram as shown by the shaded portion of *E*, calling for a later cut-off and more steam.

The shape of this end of the diagram depends largely upon the amount of expansion and consequent terminal pressure. If the

steam is expanded exactly to the line of counter pressure the diagram will terminate in a sharp point as at *F*, and at the end of the stroke the cylinder will be full of steam of the same pressure as that existing in the exhaust pipe. When the exhaust valves are opened there is, therefore, no flow, either out of or into the cylinder, except such as is caused by the movement of the piston. When the cut-off is late more steam is admitted, and has to be expelled, and we get an appearance more like *G*; and between this and the point shown at *F* there may be any variety of shapes, according to the terminal pressure and setting of the valves. When the steam is cut off so early that the expansion extends below atmospheric pressure, or the pressure against which the engine is exhausting, we get an appearance like that shown at *H*. Here at the moment of release the pressure in the exhaust pipe is greater than that in the cylinder, and when the valve is opened at *a* there is an inrush of the previously exhausted steam, raising the pressure to the counter-pressure line. This condition is apt to cause a disagreeable slamming of the exhaust valve, which is lifted from its seat when the pressure in the cylinder becomes less than that beneath the valve, and is slammed closed again when steam is admitted. It may be stopped by throttling the initial pressure so that the lessened expansion does not cause a loop.

During the formation of this loop the pressure urging the piston forward has been less than that against which the piston moves, the forward motion continuing only by reason of the momentum of the fly-wheel and moving parts, so that the area of the loop represents just so much work exerted against the piston, and must be subtracted from the other area of the diagram to get at the effective work. This point will be considered in detail when we come to working up the diagram for power.

CHAPTER IX.

THE COUNTER PRESSURE LINE.

The tendency of a piston to move depends upon the difference in pressure upon its two sides. If there were 30 pounds pressure in both ends of the cylinder at once the piston would not move any more than though there were no pressure at all. If there were 30 pounds pressure on one side and 15 pounds on the other the force with which the piston would tend to move would be the same as though there were 15 pounds on one side and nothing on the other. In other words, the "effective" pressure is the unbalanced pressure, or the difference in pressure between the two sides.

The pressure upon the piston during the forward stroke is represented by the steam and expansion lines, the pressure in the same end of the cylinder during the backward stroke is represented by the exhaust, counter pressure, or back pressure line, as it is variously called. Obviously an engine will be doing the greatest amount of work when the pressure urging the piston forward is greatest and the retarding effect of the back pressure is least. Steam will not flow, however, from one place to another without a sufficient difference in pressure to overcome the resistance to movement through the connecting pipes and passages. If at the end of the stroke the steam has been expanded to atmospheric pressure in a non-condensing engine, there will be no immediate outrush of steam from the cylinder, when the exhaust valve opens, because there is no greater pressure in the cylinder than that of the atmosphere into which the steam must flow. The steam must therefore be pushed out by the piston, and the resistance to its movement will depend upon the velocity with which it flows, and the length and directness of the exhaust pipe. The size of the exhaust pipe and passages is involved in the velocity of flow. If the exhaust pipe were as large as the cylinder

and directly open to it the rate of flow in linear feet per minute would be the same as the piston speed. If the area of the pipe or the passage leading thereto were one-half the cross sectional area of the cylinder the rate of flow would be twice the piston speed, because to get through a passage of one-half the area in the same time the steam must travel twice as fast. As the resistance to flow increases with the velocity, it is found desirable to limit the rate of flow in the exhaust passages to 6,000 linear feet per minute, which, for a piston speed of 600 feet per minute, requires for the exhaust passages a cross-sectional area of one tenth that of the cylinder. For other piston speeds the proper area of the exhaust passages may be found by multiplying the cross-sectional area of the cylinder by the piston speed in feet per minute and dividing by 6,000.

The compression of the steam by the piston pushing it out of the cylinder against the resistance to flow through the pipes and passages, will show on the indicator diagram in raising the line of counter pressure above the atmospheric line in a non-condensing engine. In a well proportioned engine at moderate piston speeds and exhausting through a short and ample exhaust pipe this moving pressure will not be noticeable with an ordinary spring, and the line of counter pressure will merge into the atmospheric line, as at *A*, Fig. 63. Under less advantageous circumstances, however, the back pressure line will be elevated above the atmospheric line, as at *B*, and the distance between them will be a measure of the force required to overcome the resistance to the outflow of the exhaust. The beginning of the back pressure line depends, as may be seen from the last chapter, very much upon the point of release and the terminal pressure. When at the end of the stroke the cylinder is full of steam of a high pressure, we have a rapid outflow of steam as soon as the valve is opened for release, but even with the greater impelling pressure a sufficient velocity is not generated to discharge this greater volume of steam (which expands when the pressure is reduced) before the piston gets some distance on its way back, making the beginning of the back pressure line like *C*; and sometimes the back pressure does not reach its lowest point until the backward stroke is practically completed, as at *D*.

Sometimes we find a diagram where the back pressure line starts in well enough but makes a gradual rise toward the center

of the diagram, falling again as the stroke is completed, as at *E*. This may be caused by too great velocity in the middle of the stroke, either from contracted ports or too much inside lap on a slide valve narrowing up the exhaust passage as the center of the stroke is reached, and where the piston, and consequently the steam, has the greatest velocity. The same effect may be produced upon a Corliss engine. It is also found where a pair of cylinders working on cranks set at 90 degrees exhaust into the same pipe, the release of one cylinder occurring practically in the

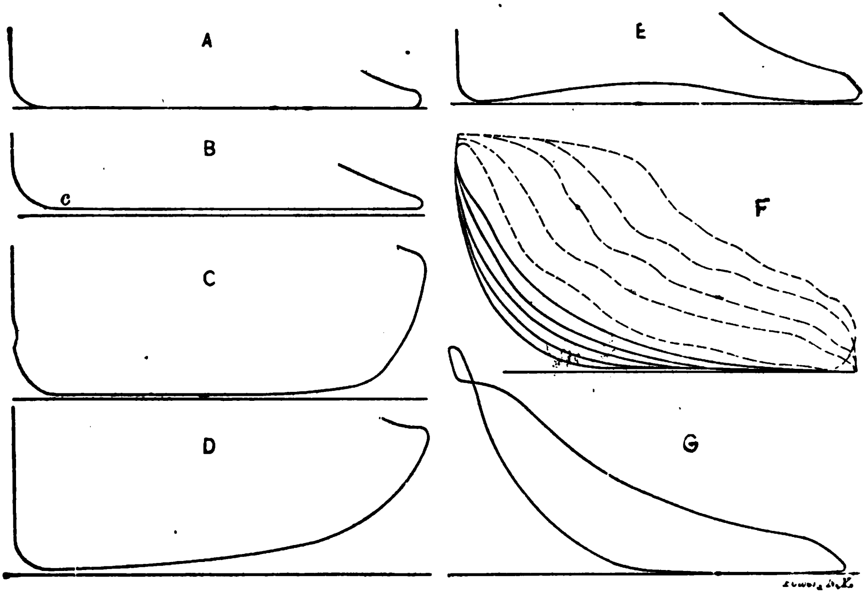


Fig. 63.

middle of the stroke of the other and the efflux of steam into the pipe caused a rise of pressure.

The end of the back pressure line depends for its shape upon the amount of compression. At *c* in diagram *B* Fig. 63, for instance, the exhaust-valve closes and the steam remaining in the cylinder is compressed, the pressure rising upon the curve shown. With no compression the back pressure line would continue straight to the end of the diagram, and with a prompt admission we should have a square corner at that end. When the compression commences earlier in the stroke the compression curve

runs proportionally higher, as is well shown at *F*, taken from an engine where the compression varies with the load, and showing the effect upon the counter-pressure line of closing the exhaust-valve at different points in the stroke. It is even possible to carry the pressure by compression above that in the steam chest, so that when the valve opens for the admission of steam, the pressure in the cylinder being greater than that in the steam chest, there is a drop instead of a rise to the line of realized pressure, as shown at *G*.

CHAPTER X.

THE COMPRESSION LINE.

Compression is the inverse or opposite of expansion. In making the expansion line the volume of steam admitted up to the point of cut-off is increased in volume, the pressure falling in an inverse ratio, and we remember that the product of the volume and pressure was constant. In compression the volume of steam enclosed when the exhaust-valve closes is diminished in volume with a consequent increase in pressure, and in this case too the product of the volume and pressure is constant. If we compress the steam into half the space which it occupies when the exhaust-valve closes we shall double its absolute pressure; into one-third the space, treble its pressure, etc. The clearance space being in most cases a large proportion of the volume inclosed becomes of increased importance.

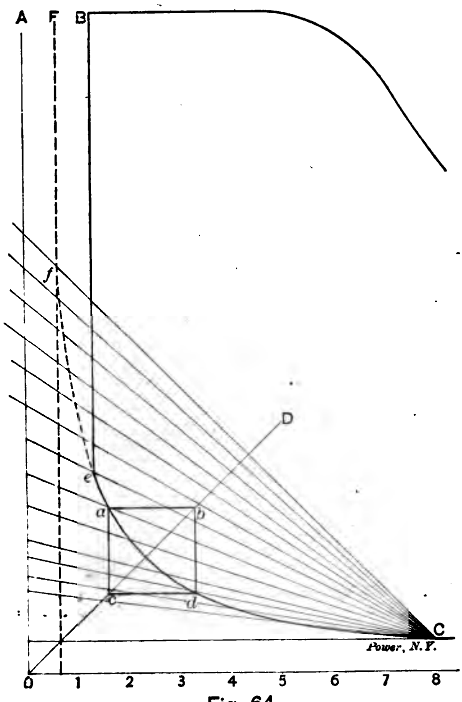


Fig. 64.

In Fig. 64 suppose the exhaust-valve to close at C and the clearance to be bounded by the line O A. There is then shut into the cylinder when the exhaust closes a volume of steam pro-

portional to the line *O* 8 and of an absolute pressure equal to 8 *C*. When the piston has advanced to 4 this volume will be one-half of *O* 8 and the pressure will be twice 8 *C*; so at 6 the volume will be 6-8 of that at *C* and the pressure 8-6; at 1 the volume will be ⅛ and the pressure 8 times that at *C*. The pressure at the various points can be calculated and measured upon the ordinates by scale, or the line can be laid out graphically for the compression line by any of the methods shown for the expansion line by using *C* in the same manner that the point of cut-off was used in laying out the expansion line, and spacing off vertically upon the line *O A*, or on an extension of 8 *C* instead of upon *O* 8, as for the expansion line. In Fig. 64 the curve is laid out by the method described in Fig. 54 page 61. It is rarely that it is of service to apply the curve to the compression of an actual diagram unless it is from a single valve automatic engine where under light loads the compression line becomes nearly as large and important as the expansion. It will be remembered that in Fig. 59 page 67 it was shown that if a rectangle was constructed upon the expansion line, with sides parallel and perpendicular to the atmospheric line, its diagonal prolonged would cut the zero line *O X* at the intersection of the line *O A* bounding the clearance. This is equally true of the compression line, and it will be seen in Fig. 64 that the diagonal *O D* of the rectangle *a b c d* cuts *O* 8 at the intersection of the clearance line *O A*. This construction is often useful for determining the amount which the diagram must be lengthened for clearance. In Fig. 64 the admission valve commences to open at about *e*, and as the piston comes to a standstill' merges the compression into the admission line. The dotted line shows where the pressure would go to if the piston advanced further into the clearance.

It is difficult for some engineers to understand how you can get compression in a condensing engine. There is, they reason, a vacuum in the cylinder when the exhaust valve closes, and you have nothing to compress. This wonld be true if the vacuum were complete, but the "vacuum" of practice is simply an absolute pressure less than that of the atmosphere. The less the absolute pressure the more complete the vacuum. The pressure of the atmosphere is equal to about 15 pounds or 30 inches of mercury. When we have a vacuum of 26 inches we

have still in the condenser an absolute pressure of 30 — 26 = 4 inches of mercury or two pounds available for compression.

The amount of pressure or the effective compression obtained by closing the exhaust valve, however, does depend upon the tension or pressure of the vapor enclosed in the cylinder when the exhaust valve closes. Referring to Fig. 65, suppose we have an engine where the clearance space *O A* is one quarter of the total volume, *O C* between the piston, cylinder head, and valves after the exhaust valve closes. If the counter-pressure line of the diagram was only 3 pounds above the line of absolute zero, corresponding to a vacuum of 24 inches, there would be three

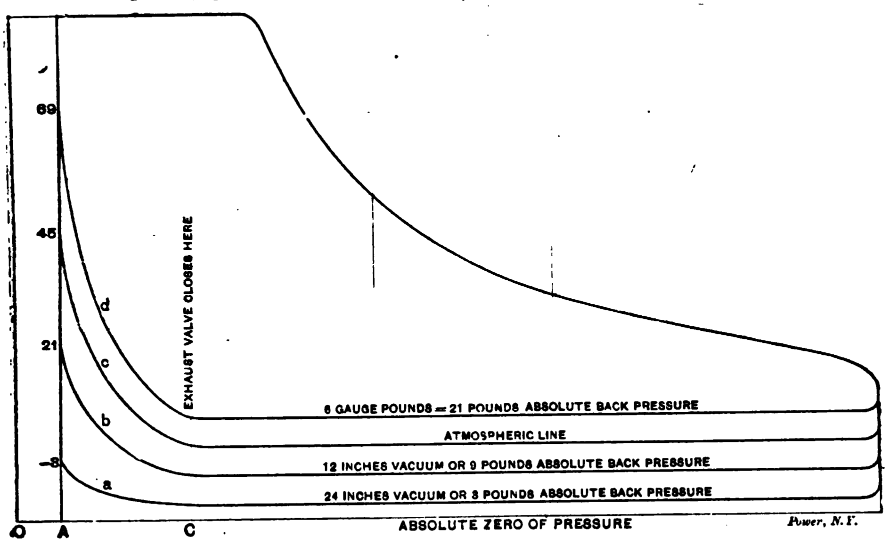


Fig. 65.

pounds less than the atmospheric pressure at the end of the stroke, as shown at *a*. If we had only 12 inches of vacuum or 9 pounds absolute to start our compression with we should get up to 21 pounds as at *b*. With a non-condensing engine and no back pressure (above the atmosphere) we should get 45 pounds by compression as at *c*, while with 6 gage pounds back pressure we should get up to 69 pounds above the atmosphere with the same valve setting and point of exhaust closure that gave us three pounds less than atmospheric pressure with the low counter-pressure line.

The smaller the clearance, too, the greater the pressure realized by compression, with the same point of exhaust closure, on account of the small final volume possible. In Fig. 64, with the clearance *A B*, we realized a pressure equal to *e*. If we had half the clearance, i. e., if the piston could have advanced to F, we should have realized a pressure equal to *f*. In engines with a variable compression it is necessary to have a considerable proportion of clearance or the pressure would be excessive with the early exhaust closure usual with light loads. As it is, the pressure generated by compression frequently exceeds the initial pressure (see diagram 6, Fig. 63, page 70.)

The object of compression is initially to furnish a cushion or gradually increasing resistance, to bring the moving parts to rest and change the direction of the push upon them without the shock which would follow upon the sudden opening of the steam valve. In Fig. 66 the piston is moving to the right, or toward the shaft, and the engine is about in the position shown in the small sketch between the diagrams.

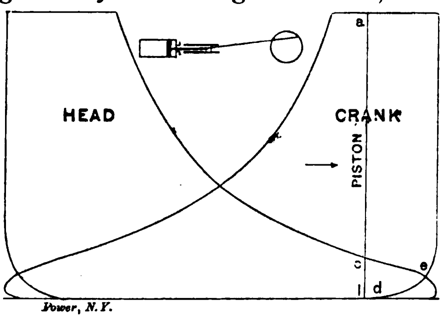


Fig. 66.

Every joint between the piston and the main crank pin is in compression, and the main shaft is pushed hard against the outer face of the bearing. When the crank reaches the center, and the pressure acts on the other side of the piston, the connecting rod will pull instead of push, every joint will be extended, and the main shaft pulled against the back of the bearing. If this change in pressure is effected suddenly every particle of lost motion in every joint and bearing will be taken up with a thump, and it is only by changing the pressure gradually from one side to the other that we can make it run smoothly. When the piston is at the point in the stroke indicated at *a l*, Fig. 66, there is behind it the pressure *l c*, and no pressure but that of the atmosphere in front of it. As it moves along, the pressure behind it decreases while at *d* the pressure in front of it begins to increase, and at *e* the pressures on both sides are equal. After this the pressure in

front exceeds that behind the piston, but the change is gradual, the direction of thrust is changed under a slight difference of pressure, and when the steam is admitted the bearings and journals are already firmly pressed against the surfaces upon which they are to bear.

Aside from its cushioning effect compression has another advantage in reducing the loss from clearance. Take an exaggerated instance. Suppose we had an engine with a clearance equal to 100 per cent, i.e., that the volume of steam required to fill the space behind the piston, including ports, etc., when the engine is on the center, is equal to the volume generated by the piston's movement, i.e., the piston area multiplied by the length of the stroke. It is understood that the indicated power is in proportion to the inclosed area of the diagram. Before the piston can move, the clearance must be filled with steam up to initial pressure, and supposing the engine to work without expansion it would take two cylinderfuls of steam to do the work of one stroke, one to fill the clearance, and one to supply the space behind the moving piston.

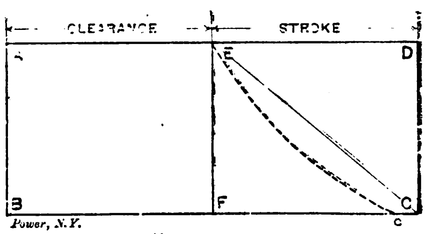


Fig. 67.

In Fig. 67, then, we should require a volume of steam proportional to the rectangle *A B C D* to do an amount of work proportional to the rectangle *E F C D*. Now suppose we close the exhaust valve at *c* so as to fill the clearance by compression with steam at the initial pressure, we have reduced the area of the diagram by the amount below the dotted line, but we have still considerably *more than half* of it left, and as the clearance is already full, have used *only half* the volume of steam.

Where there is no expansion the steam required to fill the clearance space is a dead waste. With a cut-off engine it gets a chance to expand with the other steam and does some good, but still there is a saving by compression and theoretically the greatest area of diagram will be produced by a given volume of steam when the ratio of compression equals the ratio of expansion, i.e., when the clearance bears the same relation to the volume at the

commencement of compression, that the volume of cut-off does to the volume at the end of the stroke. In Fig. 68 we have a diagram where the clearance-space OA is 25 per cent of the volume up to cut-off, and the number of expansions OX divided by OF is 4. The shaded portion of the diagram would give 56 pounds mean effective pressure. Suppose that,

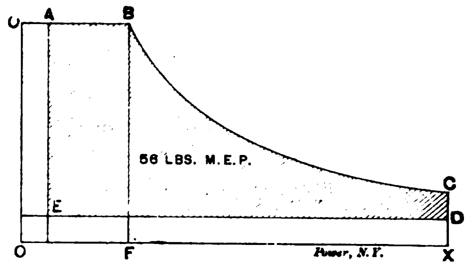


Fig. 68.

as in Fig. 69, we compress up to initial pressure, we cut out the area *abc*, i.e., we expend 10.6 pounds mean effective pressure in filling the clearance, but we have $56 - 10.6 = 45.4$ pounds left, and have used only three-quarters of the steam. Three-quarters of 56 is 42, so we have gained 3.4 pounds.

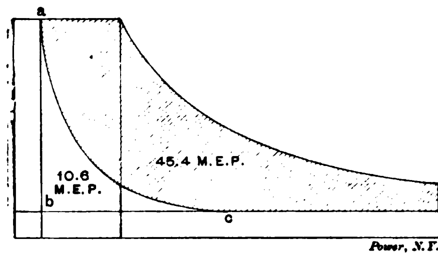


Fig. 69.

It is not found in practice that the best results follow from compressing to the boiler pressure any more than they do from expanding to the back pressure. In Fig. 70 we have a diagram where the steam is cut off at such a point as to expand just to the line of counter pressure, and where compression just reaches initial pressure, giving theoretically the greatest amount of mean effective pressure, or diagram area at the least consumption of steam. But the mean effective pressure from such a diagram would be very small, 13.4 as it is plotted. An engine of given size would give a small amount of power under such conditions, while the loss from friction and cylinder condensation

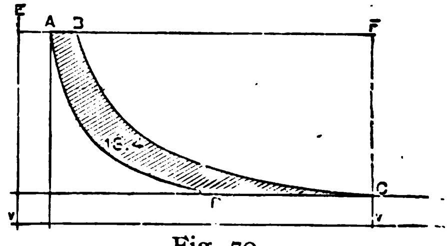


Fig. 70.

would be as great as though the load were larger. The loss per horse-power would be greater than the gain, and it is found to be more economical to cut off further in the stroke, as in Fig. 71, which, though it involves the loss of the area *CKM* in capability for expansion in the steam, increases the effective area of the diagram so much that the mean effective pressure is raised to

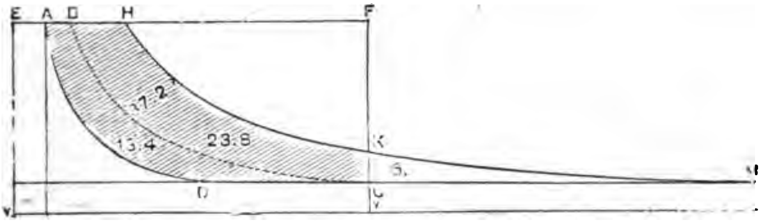


Fig. 71.

37.2 with no increase of the condensation and friction losses. If the cut-off were carried out further the area gained becomes smaller as we get up into the corner at *F*, while the waste area *CKM* increases rapidly. Just as it is found advisable to increase the area of the diagram at the expense of some free expansion at the end of the stroke, so it is found advisable to add to it by reducing compression, at the expense of not quite filling the clearance. Fig. 72 shows how by incurring a loss due to free expansion in the clearance proportional to the area *PAO*, the area of the diagram is increased to 47.1, and though the volume

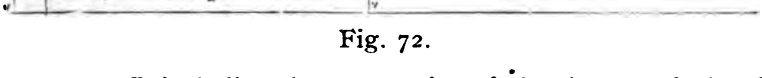


Fig. 72.

up to cut-off, including that proportion of the clearance inclosed in *PAO* would bear a greater proportion to the shaded area of the diagram than *AB* does to the area of Fig. 71, the steam actually required per horse-power will be less on account of the less proportional loss from condensation and friction. Mr. F. H. Ball, of the Ball Engine Co., from whose argument the last three

diagrams are copied, contends that "if in a given diagram any free expansion takes place at the terminal pressure, then compression should not rise to initial pressure, but the compression curve should be removed from the curve of full compression a sufficient distance, so that the useful work thereby added to the diagram shall be accompanied with the same proportionate loss by free expansion that is found to accompany the work that has been added to the full expansion curve." ("Full compression" is here used to describe the curve which just reaches initial pressure within the vertical lines representing the piston travel, as $D\ A$, Fig. 70. The term "full expansion curve" is

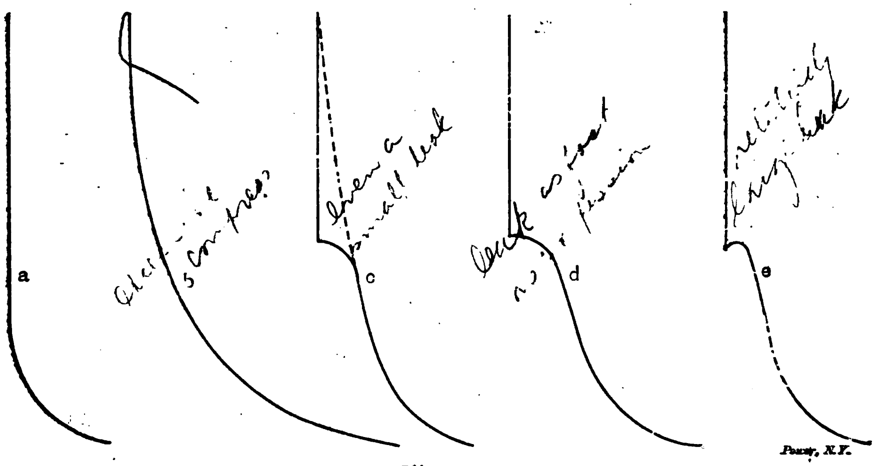


Fig. 73.

used to describe a continuous expansion curve, terminating on the line of back pressure within the limits of piston travel as $B\ C$, Fig. 70.)

This furnishes a good reason why we do not and should not compress to initial pressure. On the other hand, we must compress enough to fulfill the initial object of compression, the smooth running of the engine.

Little remains after our consideration of this line as presented in the foregoing text except to consider some of the forms obtained in practice. When the engine is of a type in which the compression is constant, the best results will be attained under normal loads by having the compression round up nicely into the

admission line, as at *a*, Fig. 73, meeting the perpendicular line at about one-third of its height. This will require a different setting of the exhaust valve for different heights of the counter pressure line, as explained on page 79, and can only be determined by the indicator. If no indicator is used, put on only enough compression to make the engine run smoothly. At *b* is shown excessive compression, the pressure running up above that in the steam chest, so that when the valve opens for admission, steam flows from the cylinder to the chest and the pressure falls. A form of compression line often met which is shown at *c*, where the pressure instead of continuing upward along the dotted curve falls away as shown. When this occurs we must look for some cause for the reduction of pressure, and we will usually find it in a leak. As the piston approaches the end of its stroke its movement becomes very slow, the volume of steam involved is small and growing smaller, and if there is even a slight leak in the exhaust valve, drip valve, or piston there will come a time when the volume of steam discharged through the leak will equal the volume generated by the movement of the piston in the same time.

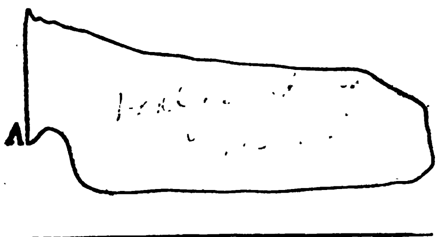


Fig. 74.

To state it more simply, at all times the pressure will be lower than if there were no leakage, and there will come a time when the escape through the leak with the increasing pressure will pull the pressure down as fast as the movement of the piston increases it, and the line will become horizontal as at *d*, or it may even fall away as at *e*. As soon as the pressure, from compression, behind the piston becomes greater than that in front of it a leak in the piston becomes effective to reduce the compression pressure. Such a diagram as Fig. 74, which was sent to the author for explanation as to the formation at *A*, might be caused by a badly leaky piston. It will be seen that the compression rises after the valve closes much more abruptly than it should have done at that distance from the end of the stroke. This would be accounted for by leakage from the other side, where the pressure is still high, into the confined space in front of the

piston. As the pressure behind the piston decreases, this action falls off, allowing the line to lean, and after the release occurs on the other end the leak is reversed, from the compression space into the other end, now opening to the exhaust, allowing the pressure to fall off as shown. It is probable, as the exhaust closure is early and the release late on this diagram, that a diagram from the other end of the cylinder would show opposite conditions, early release and little compression, which would locate the turn in the curve about where it occurs in the diagram. As a general rule, when you see a compression line falling off badly, look out for leaks. It is a better indication than a failure of the expansion line to follow the theoretical.

It is a matter for consideration, however, if condensation does not play an important part in the formation of such departures from the regular curve. The surfaces of the cylinder head, piston, and ports have just been exposed to the temperature of the exhaust, and as the piston nears the end of its stroke they bear a large porportion to the small volume of steam inclosed. Enough steam must be condensed upon those surfaces to bring them up to the temperature corresponding to the pressure before the steam can remain as steam in contact with them, and this condensation might account for the slight falling off in pressure necessary to produce the slighter of these deviations from the true curve.

CHAPTER XI.

MEASUREMENT OF THE DIAGRAM FOR MEAN EFFECTIVE PRESSURE.

One of the principal uses of the indicator diagram is to determine the horse-power which the engine is developing. One of the important factors in this problem is the pressure urging the piston forward, and this can be found with any accuracy only from the indicator diagram. The pressure varies through the

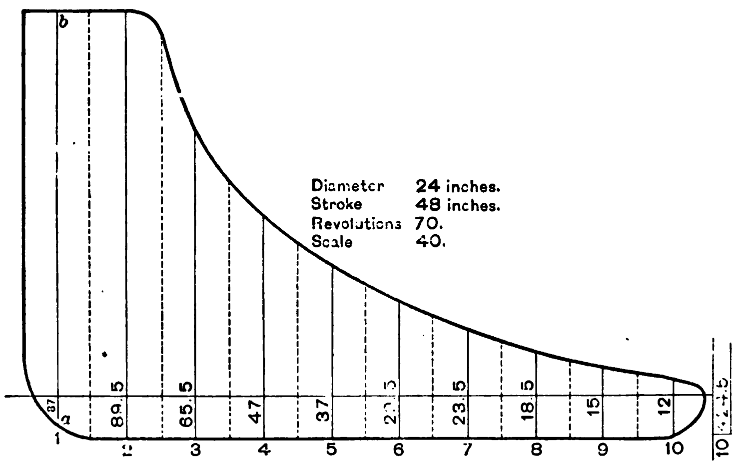


Fig. 75.

Power, N.Y.

stroke, and is opposed by a varying amount of back pressure, so that the average unbalanced, or as it is commonly called, the "mean effective pressure," must be determined. The most elementary way of doing this is by measuring the pressure upon the diagram at a number of equidistant points and taking the average. To do this, divide the diagram into a number of equal parts lengthwise, ten for ordinary work, as shown in Fig. 75 by

the dotted lines. Now with a scale corresponding to the spring with which the diagram was taken, measure the pressure in the center of each of these divisions; that is, upon the full lines or ordinates. Notice that this pressure must be measured between the lines of the diagram, as from *a* to *b*, whether the engine is condensing or non-condensing, and not from the atmospheric or any other line.

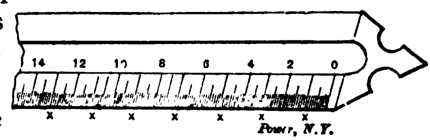


Fig. 76.

Performing this operation on the diagram shown in Fig. 75, we find, with a 40-pound scale, 87 pounds on the line or "ordinate" 1, 89.5 pounds on 2, 65.5 on 3, 47 on 4, 37 on 5, 29.5 on 6, 23.5 on 7, 18.5 on 8, 15 on 9, and 12 on 10. Adding these values we have 424.5 for the sum, and dividing by 10, the number of measurements, find the average or mean effective pressure to be 42.45 pounds.

Several expedients may be resorted to for shortening the labor of dividing the diagram and locating the ordinates. The simplest of these is to have a rule, a little longer than the ordinary

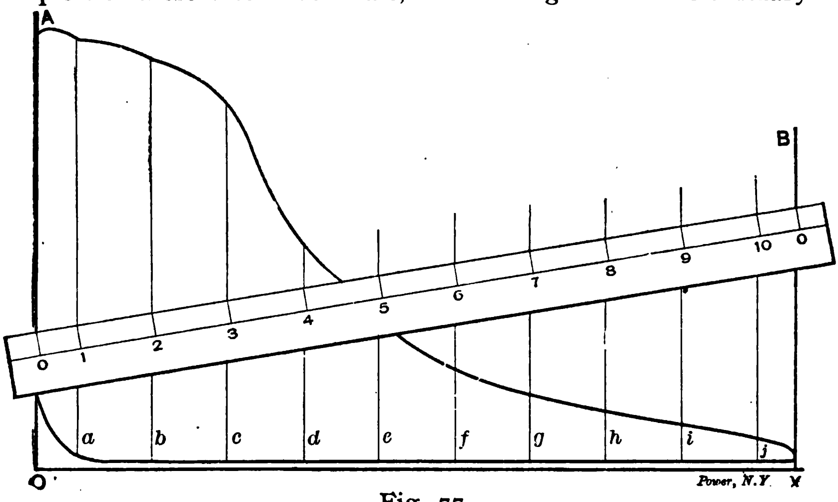


Fig. 77.

length of your diagrams, divided as shown in Fig. 76, just as you want your diagram to be divided, with nine spaces of equal length in the middle, the two end spaces, 0 to 1 and 10 to 0, being one-half the width of the others. Four inches between-

the zero marks is a good length for diagrams from 3½ to 4 inches in length, and one each of 3½ and 4½ inches, with a short one for the diagrams from small cylinders, will cover all ordinary cases.

Draw the lines *O A* and *X B* at the extreme ends of the diagram and perpendicular to the atmospheric line. Place the rule between them, as shown in Fig. 77, at such an inclination that both zeros come upon the perpendiculars. Then with a needle-point prick the card opposite each division of the rule, and draw the ordinates perpendicular to the atmospheric line and through these points. An engineer's scale, such as shown in Fig. 76, may be used to good advantage in this work. It has places for

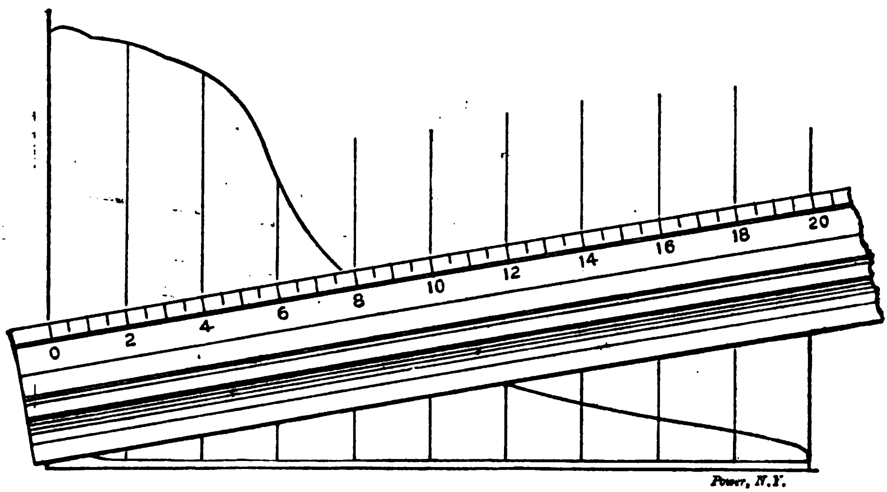


Fig. 78.

six scales, and these, instead of being feet and inches in the various proportions of the ordinary draftsman's scale, are plain 10ths, 20ths, 30ths, etc., of an inch, which adapts them for use in measuring diagrams taken with the corresponding scales. It is better to get a scale from 20ths to 80ths, as the 20 scale can be readily used to measure for a 10 spring by calling the divisions half pounds. Such scales are sold generally by dealers in drawing instruments, the price in New York being eighty cents for a scale six inches in length, and a dollar and a half for one twelve inches long. They are also made in 14 and 18 inch

lengths. The six inch length is admirable for carrying in the indicator box. To use this, erect perpendiculars at the extremes of the diagram, just as in the previous method.

If the diagram is just 4 inches long the 20 pound divisions of the 50 scale will just divide it into ten equal parts. If it is less than four inches incline the scale as in Fig. 78, so that the zero is upon one line and the 20 on the other. The figured divisions will divide the space into ten equal parts. In order to get a half space on each end, that is to locate the ordinates in the center of the equal tenths, slide the scale to the position shown in Fig. 79 so that the 1 mark is on one line and the 21 mark on the

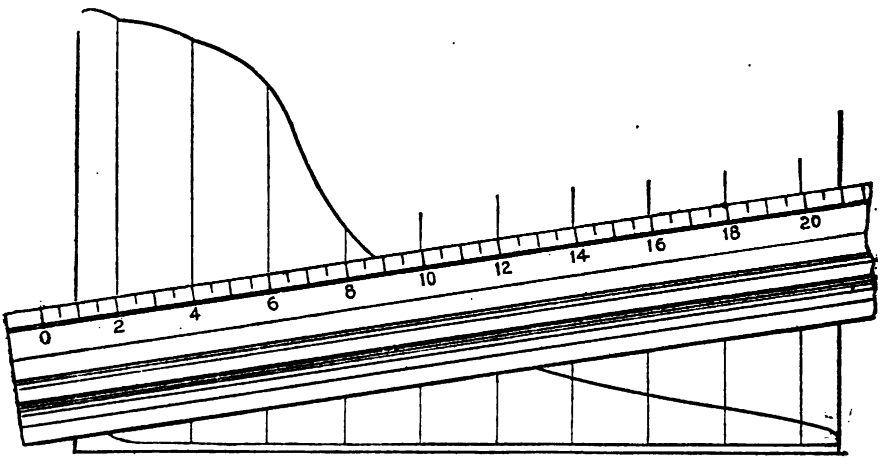


Fig. 79.

other. Make a needle hole or pencil mark at the edge of the scale against each numbered division and erect the ordinates square with the atmospheric line and passing through the points indicated. The 50 scale works very well down to diagrams 3⅓ inches in length, which are exactly divided into tenths by the numbered divisions of the 60 scale; and for this length and below, the 60 scale will preferably be used, as the inclination of the 40 scale becomes too great. For diagrams between 4 and 5 inches the 40 scale is used in the same way. No calculation is required. If the diagram is, on trial, too long for the 50 scale, use the 40; if you have to use the 50 scale at too much of an

angle, use the 60. A very little use will make the process perfectly natural.

The principal advantage of such a scale, however, especially the 12 or 14 inch scale, is in measuring the length of the ordinates. Usually the pressure on each ordinate is measured with the minute divisions of the common scale, the ten observations added, and the sum divided by ten to get the average. Now we can divide by ten to start with by dividing the value of our scale and at the same time get the advantage of the coarser reading. With a 40 spring, instead of calling 1 inch 40 pounds, suppose we call it 4 pounds. Then we can measure the ordinate,

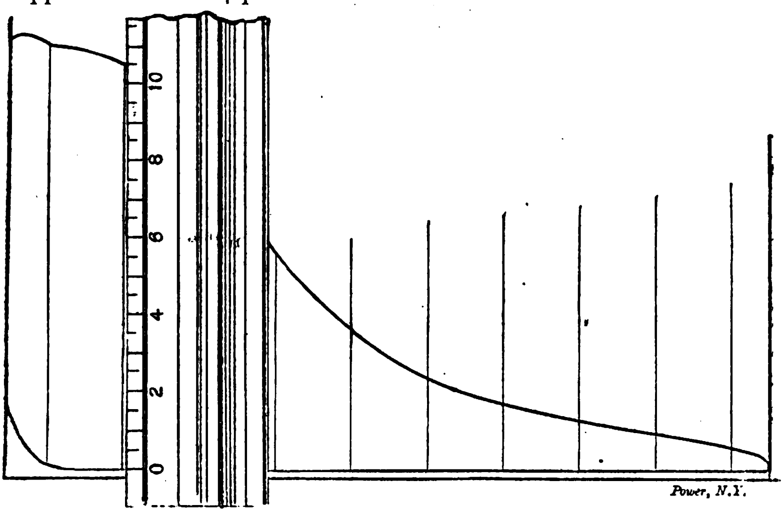


Fig. 80.

add the results, and have our mean effective pressure at once. The pound instead of being $\frac{1}{40}$th of an inch will be ¼th. The finest divisions of the scale will represent tenths of pounds instead of full pounds, so that we can read much more accurately, and the numbers on the scale will correspond with the pound marks. Thus in Fig. 80 we have on the ordinate to which the scale is applied, 10.5 pounds pressure. This is, of course, only one-tenth of the pressure which that particular ordinate represents, but we shall give the pressure ten records, so that the aggregate will be the same as though we measured each on the given scale and then divided the aggregate by ten.

There are also procurable from the instrument makers parallel rules, as shown in Figs. 81 and 82, whose method of application is too obvious to require description.

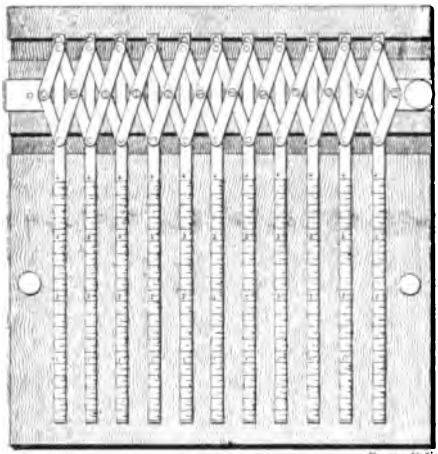


Fig. 81.

Instead of measuring each ordinate with the scale corresponding to the spring with which the diagram was taken, some engineers prefer to lay off the lengths of the ordinates continuously on the edge of a strip of paper, then to either measure the whole length with a long scale of the proper unit, or with a scale of common inches, and multiply the length by the scale of the spring.

In the measuring of the mean effective pressure by ordinates there remains to be explained the treatment of cards having negative or back pressure areas. For example, in Fig. 83. after the point *a* is passed, the forward pressure in the cylinder is less than the back pressure during the return stroke. The piston is actually hanging back upon the engine, and the loop not only represents no addition to the useful mean effective pressure, but a force acting against the motion of the engine equivalent to so much back pressure. The average pressure of the loop portion of the diagram must therefore be subtracted from that of the other portion. Erecting the ordinates

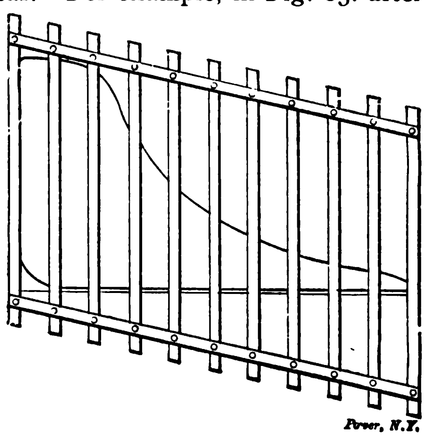


Fig. 82.

as before directed, and measuring with a 40 scale, we have $98 + 93 + 40 + 20 + 5 = 256$ as the sum of the measurements in the main portion of the diagram, and $3 + 8 + 13 + 15 + 11 = 50$ as

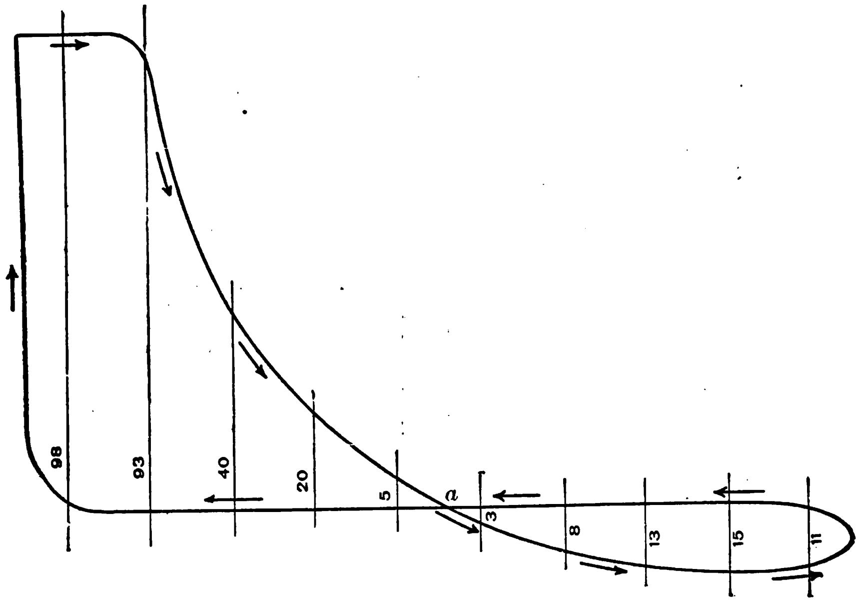


Fig. 83.

the sum of the measurements in the loop. Taking the difference and dividing by 10 to get the average, we have

$$\frac{256-50}{10} = 20.6 \text{ lbs. M. E. P.}$$

CHAPTER XII.

THE PLANIMETER.

The area of a rectangle, as *A*, *B*, *C*, *D*, Fig. 84, is found by multiplying its height by its length. If the figure shown were 2 inches high and 4 inches long it would obviously contain 8 square inches of area. If on the other hand we knew that its area was 8 square inches and its length 4 we could easily tell that it was $8 \div 4 = 2$ inches high. If we wanted to know how high it would be if it were any other length to contain the same area, we would simply divide the area by the new length. If the rectangle in Fig. 84 were lengthened to 8 inches it could, to contain the same area, be only $8 \div 8 = 1$ inch high, or if lengthened to 6 inches $8 \div 6 = 1\frac{1}{3}$ inches.

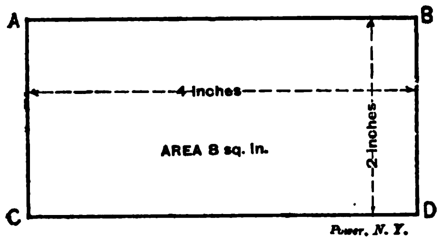


Fig. 84.

Suppose now we have a figure like Fig. 85, and wish to know its average height. We could divide it into a number of rectangles, as shown by the dotted lines, and find the height which each rectangle would be if extended to the full length of the diagram. Supposing the diagram to be 4 inches long, the area *A* would be one-half an inch high and an inch long, containing therefore one-half a square inch of area. If this were extended to 4 inches its height

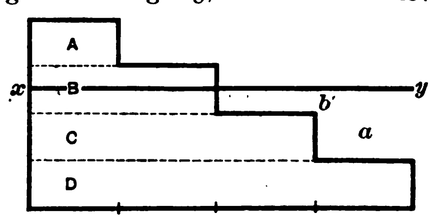


Fig. 85.

would be reduced to $\frac{1}{2} \div 4 =$ ~~one-fourth~~ 1/8 of an inch. The area *B* is $2 \times \frac{1}{2} = 1$ square inch, and would be $1 \div 4 =$ one-fourth of an inch high if 4 inches long. Similarly *C*, containing $1\frac{1}{2}$ square inches, would be $1\frac{1}{2} \div 4 =$ three-eights of an inch high, and *D*, already 4 inches long, is one-half an inch high. So for the total average height we should have $\frac{1}{8} + \frac{1}{4} + \frac{3}{8} + \frac{1}{2} = 1\frac{1}{4}$ inches, bringing the average height at the line *xy*. That this is right is evident at a glance, for the area *A* will just fill the space *a*, and that part of *B* which is above the line *xy* will just fill the space *b* below it. But if we know in the first place the area of the whole figure we can get at the average height at once by dividing that area by the length, for obviously the whole is equal to the sum of all its parts, and we shall get the same result by dividing the whole area by 4 as by dividing each of its parts by 4 and adding the quotients. Thus the whole area of Fig. 85 is $\frac{1}{2} + 1 + 1\frac{1}{2} + 2 = 5$ square inches, and $5 \div 4 = 1\frac{1}{4}$, the same as the sum of the several divisions.

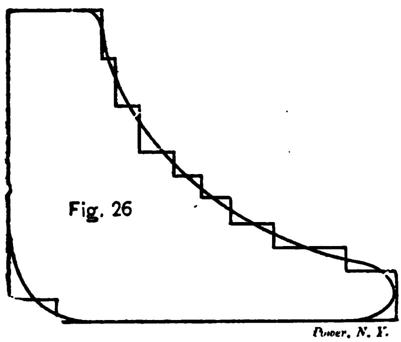


Fig. 86.

In an indicator diagram the height is proportional to the pressure, and to find the average pressure we must find the average height. We have an irregular figure which we wish to reduce to a rectangle of the same area and to know the height of the rectangle. Imagine the diagram stepped off into the boundaries of rectangles, as in Fig. 86, and it will be clear, in view of what has been said about Fig. 85, that dividing its area by its length will give its average height; and inasmuch as this is true however fine the divisions or steps, we may imagine them to be so fine as to be included in the width of the line which bounds the diagram, and arrive at the fact that the area of an indicator diagram, or any other plane figure for that matter, divided by its length equals its average height.

Fortunately a means is at hand for easily and accurately measuring the area of such diagrams. The planimeter, the instrument used for this purpose, is made in a variety of forms, and is sold

at prices ranging from five to thirty-five dollars. The Amsler was the first upon the market, and as a typical example is shown in Fig. 87. It consists of two arms pivoted together at the top, upon one of which is carried a roller free to revolve upon an axis parallel to the arm itself. The roller is divided circumferentially into ten equal parts, each of which represents a square inch of area, and each of these parts is further divided into equal parts representing each one-tenth of a square inch, as shown in Fig. 88. Close to the edge of the roller is a stationary plate having the same curvature and containing a vernier made by dividing a space nine-tenths as long as one of the large divisions of the roller into ten equal parts. In Fig. 89 let the space between *A* and *B* represent one of the larger divisions of the wheel, and the space between *C* and *D* the vernier.

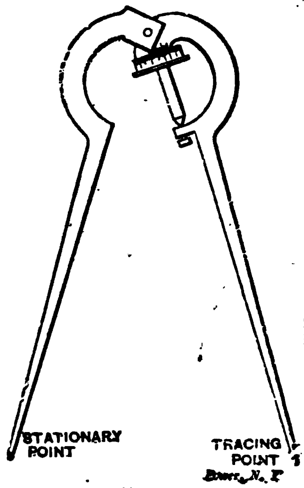


Fig. 87.

In reading the instrument we take the number on the wheel which has passed the zero mark of the vernier when the wheel is turning to the left as indicated by the arrow, as the number of whole square inches, in this case 6. The tenths of a square inch are indicated by the number of spaces, such as *a*, which have passed the zero mark, in this case 1; so that the reading of the scale as laid down in Fig. 89 is 6.1 square inches. Since the vernier *C D* is nine-tenths as long as *A B* each division of the vernier must be nine-tenths of each division of the scale. From *O* to 1 on the vernier is nine-tenths of the space beneath it on the wheel, then the space between the line *b* on the wheel and the line 1 on the vernier is just one-tenth of one of the spaces such as *a* upon the roller, the space between the lines 2 and *c* is just two-tenths, between

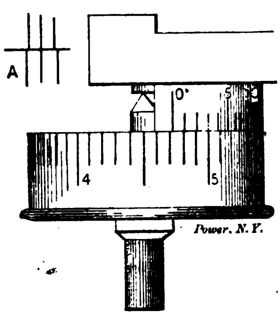


Fig. 88.

3 and *d* three-tenths, etc. If, then, the wheel rolls in the direction of the arrow one-tenth of one of the spaces *a*, corresponding to an area of one one-hundredth of a square inch, the lines 1 and *b* will coincide, for two one-hundredths 2 and *c* would coincide, so that we get the hundredths of a square inch by writing that number on the vernier which is opposite any line on the wheel. For instance, in reading the instrument as it stands in Fig. 88 we write, first, the number on the wheel to the left of the zero mark, in this case 4; then the number of whole spaces between that number and the zero mark, in this case 7; and last the number on the vernier which is in line with a mark on the wheel, in this case 3. The whole reading therefore is 4.73 square inches, the decimal point being placed after the 4, the 7 and 3 being tenths and hundredths as before explained. It will be noticed that only the zero, 5, and 10, are numbered on the vernier in Fig. 88, and this is the case in the actual instrument, the intermediate marks being easily known by their position.

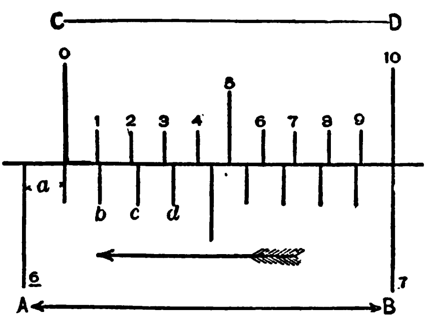


Fig. 89.

The eye soon becomes accustomed to quickly determining the mark upon the vernier which coincides with one upon the wheel, the marks at either side of it being just within the marks upon the wheel, giving the arrangement shown at *A* in Fig. 88.

The planimeter should be used upon a smooth but not slippery surface, such as that of heavy drawing paper or Bristol board. Place a sheet of this large enough to include the planimeter and the diagram upon the drawing board, and fasten it with thumb tacks. Set the stationary point of the planimeter into the paper in such a position that the tracing point can be carried around the outline of the diagram without bringing the wheel in contact with the edge of the paper. The instrument can be worked to the best advantage when it is neither allowed to close up too closely, as in Fig. 90, nor to extend too widely, as in Fig. 91. A better position for the stationary point than either of these is

shown in Fig. 92, the motion of the roller being easiest when the arms are near a right angular position. When the areas to be measured are large, or when there is considerable space between the top of the diagram and the top edge of the card, contact of the roller with the edge of the card may be avoided by inverting the diagram, as indicated by the dotted diagram in Fig. 92, using the planimeter always in the same direction, that in which the hands of a watch run; for obviously the area of the diagram remains the same in whatever position the card is placed.

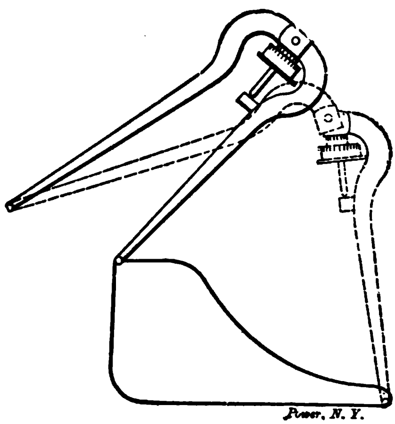


Fig. 90.

Place the tracing point on any convenient point in the line of the diagram and by pressing upon it make an incision, to mark the point of starting. Take the reading of the instrument as it stands, then with the tracing point follow the line of the diagram in the direction in which the hands of a watch move, as indicated by the arrows in Figs. 93 and 94. Follow the line as made by the pencil, not necessarily in direction (for on a right handed diagram, as in Fig. 93, you will have to trace in the opposite direction from that of the pencil which made it, in order to carry the tracing point in the direction of the hands of a watch) but in course. For instance, in Fig. 83, do not leave the expansion line at *a* and run out on the back pressure line, but follow the diagram naturally all the way around, as the arrows indicate,

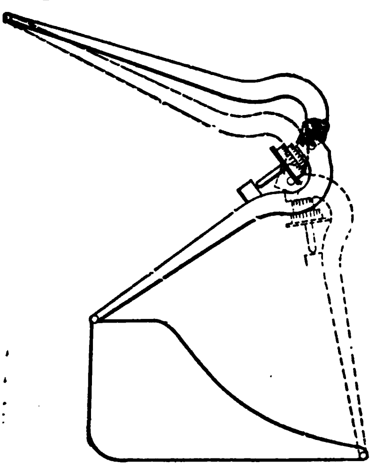


Fig. 91.

and as it was drawn by the pencil; and in Figs. 93 and 94 do the same, although in this case you will trace the diagram backward from the direction in which the pencil went over it. If the pointer traces in the opposite direction to the hands of a watch the wheel will take out the area instead of adding it. In Fig. 83 we saw that the area of the loop was negative, and that it needed to be subtracted

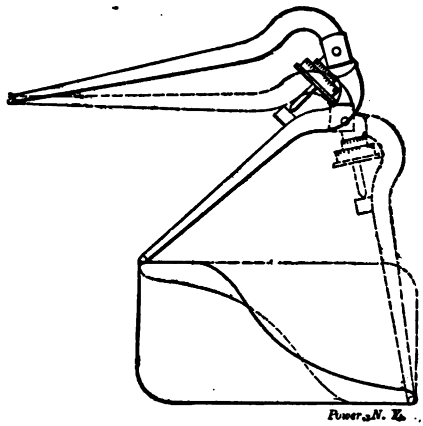


Fig. 92.

from the other part of the diagram to get the mean effective pressure. It will be seen that by following the lines of the diagram as directed the tracing point of the planimeter will pass around the negative portions of the diagram in a direction

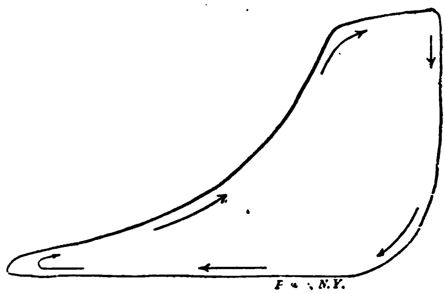


Fig. 93.

contrary to the hands of a watch, and that therefore these areas will be automatically subtracted. In this connection, be careful when starting to trace a diagram with loops, to move the tracing point in a direction that will carry it with the hands of a watch over the main portion of the diagram. If Fig. 94 for instance, were started at the point *a* or anywhere within one of the loops, the first movement of the tracing point would have to be in

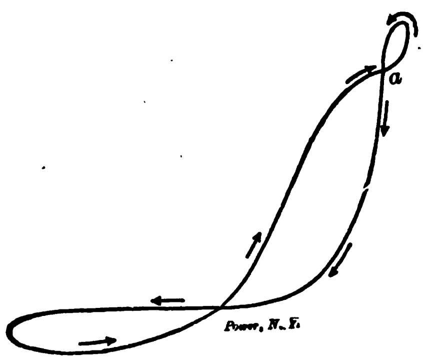


Fig. 94.

the opposite direction from that of the hands of a watch.

Having traced around the diagram and brought the pointer around and into the hole from which it started, take the reading in the new position, subtract from it the reading in the starting position, and the difference will be the area of the figure traced. If the roller were placed at zero to start with, the reading would give the area at once; but it is easier to take the instrument as it stands and subtract the initial reading. Suppose we start with the wheel at 1.42, and after tracing the diagram find the reading to be 4.69, then the area will be $4.69 - 1.42 = 3.27$ square inches. Now, to prove our work, trace the diagram again, write the result above the former reading, again take the difference, and if our work has been accurate the last reading should be 7.96. If we run around still again the reading would be 1.23. This value would really be 11.23, as we started from 7.96 and added 3.27 inches, but as the capacity of the wheel is limited to 10 inches, we have to understand the addition in the tens column and simply borrow one when we subtract the 7.96. The readings are as follows:—

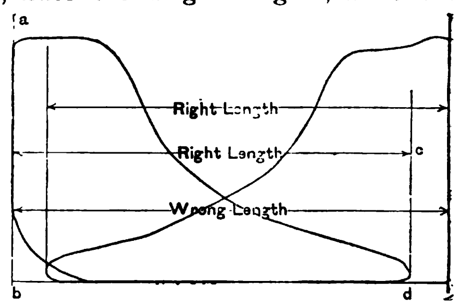


Fig. 95.

$$
\begin{aligned}
&1.23\\
&7.96 = 3.27\\
&4.69 = 3.27\\
&1.42 = 3.27
\end{aligned}
$$

The three readings agreeing, we may feel certain that our work has been correctly done and that the area of the diagram is 3.27 square inches. By dividing this area by the extreme length we find the average height.

To measure the length of the diagram, draw lines as *ab*, *cd*, Fig. 95, perpendicular to the atmospheric line and touching the extreme end of the diagram. No matter what the shape of the diagram may be, no portion of its line must extend outside of these perpendiculars, which must, however, touch the diagram at both ends. When two diagrams are taken on one card, how-

ever, remember that you want the length of *each* diagram, not the extreme length between both, as shown in Fig. 95. Now measure the horizontal distance between these vertical lines. This is very handily done with the 50 scale of the six-inch triangular scale (Fig. 76, page 88), each 50th being equivalent to .02, so that the length may be expressed direct in decimals.

Divide the area as found by the planimeter by the length, and multiply the quotient by the scale of the spring with which the diagram was taken. The product will be the *mean effective pressure.*

In a planimeter the length of the tracing arm multiplied by the movement of the wheel equals the area traced. If in Fig. 96 the length of the tracing arm (the distance between the tracing point and the hinge) is 4 inches, the circumference of the roller must be 2.5 inches in order that one revolution may equal 10 square inches. Inversely the wheel movement equals the area divided by the length of the tracing arm. If with the wheel having a circumference of 2.5 inches we used a tracing arm 5 inches long instead of 4 inches, in tracing an area of 10 square inches the wheel would not turn a full revolution. Its circumferential movement would have to be only 2 inches in order that that movement multiplied by the length of the arm might still be equal to the area, 10. The movement of the wheel, and thus the reading, is inversely proportional to the length of the arm. If the length of the arm is doubled the reading will be halved. If we make the arm one-third as long the reading will be three times as large, etc. It has been explained that to get the mean effective pressure the area must be divided by the length of the diagram. If the diagram were twice as long, with a given area the mean effective pressure would be half as much. In other words the mean effective pressure varies inversely as the length of the diagram. Since the reading varies inversely as the length of the arm, and the mean effective pressure varies inversely as the length of the diagram, we can, by making the length of the arm equal to the length of the diagram, make the reading proportional to the mean effective pressure. Suppose an instrument with an arm of 4 inches and a wheel having a circumference of 2.5 inches. One revolution of the wheel will mean 10 square inches. Suppose it is applied to a diagram 3 inches long and registers 3.75 square inches area. If the diagram was taken with a 40 spring the mean effective pressure would be

$$\frac{\text{Area} \times \text{Scale}}{\text{Length}} = \frac{3.75 \times 40}{3} = 50 \text{ lbs.}$$

Suppose now we adjust the length of the arm so that it equals the length of the diagram, 3 inches, the reading will then be $\frac{4}{3}$ds of what it was before or $\frac{4 \times 3.75}{3} = 5.00$ and by shifting the decimal point we have at once 50 pounds. Changing the length of the arm performed mechanically the division before required. For a 40 scale therefore this instrument will give us at

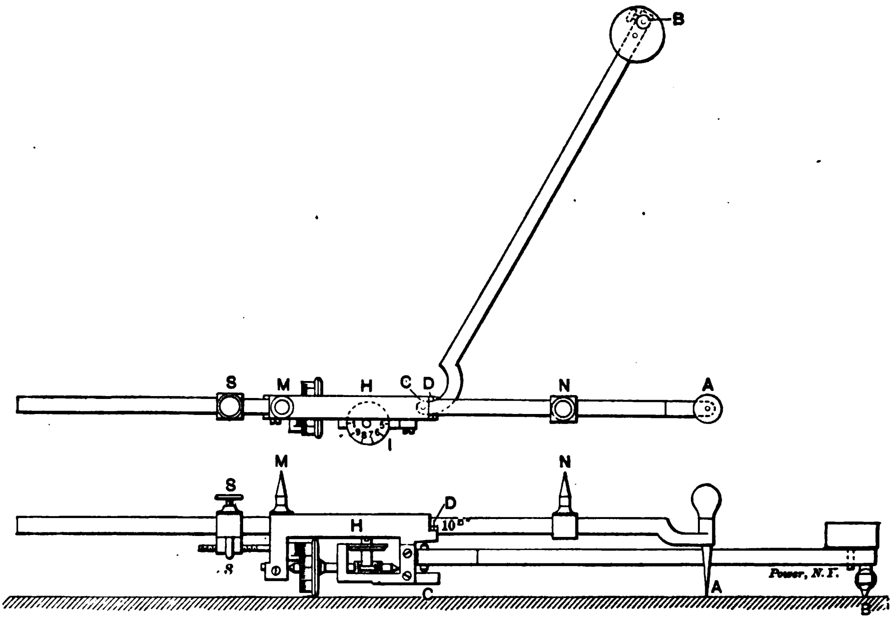


Fig. 96.

once on the wheel the mean effective pressure and for other scales the pressure can be taken proportionally one-half for a 20 scale, three-fourths for a 30, five-fourths for a 50, etc. An Amsler planimeter with an adjustable tracing arm is shown in Fig. 96. The length of the diagram is taken between the two points *M* and *N* which are always the same distance apart as the tracing point *A* and the joint *C* upon which it hinges. In another type of planimeter the reading is indicated by the sidewise movement of the wheel read against a contiguous scale as in Fig. 97 or upon the shaft upon which it slides as in Fig. 98. As these

scales are changeable and the arms adjustable the mean effective pressure can be read direct for any scale or length of diagram. Mr. Willis also provides an apparatus and set of tables by means of which his planimeter can be set to read in horse-power direct

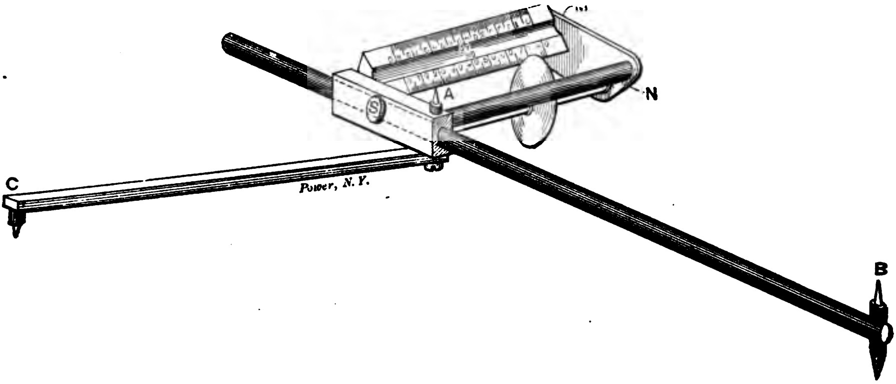


Fig. 97.

without any calculation. The instrument shown in Fig. 96 can be set to read directly in horse-power by making the length of the arm equal to

$$\frac{\text{Length of Diagram} \times 40 \times 33000}{\text{Scale} \times \text{Revs. per min.} \times \text{Area} \times \text{Stroke.}}$$

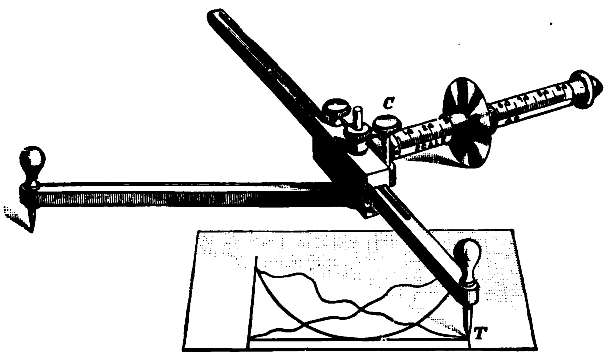


Fig. 98.

Instruments of the Willis and Lippincott class, in which a scale corresponding to that of the diagram can be used to measure the wheel movement, can be set to read directly in horse-power by making the length of the tracing arm equal to

$$\frac{\text{Length of Diagram} \times 33000}{\text{Revs. per min.} \times \text{Area} \times \text{Stroke.}}$$

If this gives an impracticable length of arm the required length can be multiplied or divided by a number which will make it practicable and the reading multiplied or divided by the same number. If, for instance, the formula called for an arm of 1.5 inches it would be better to have the arm 3 inches and multiply the reading by 2.

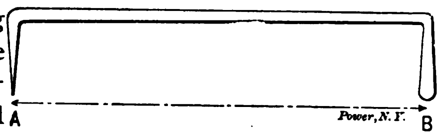


Fig. 99.

A home made planimeter with which it is possible to do quite accurate work may be made by bending a piece of wire as in Fig. 99, flattening and sharpening into a knife-edge the end at *B* and pointing the end at *A*. The distance *A B* should be 10 inches.

Locate roughly, by judgment, the geometrical center of the figure, its center of gravity, so to speak, the point upon which it would balance if cut out of cardboard as in Fig. 100. In the indicator diagram, Fig. 101, this point would be at about *A*. Draw the line *A B* connecting the center with any point upon the circumference, set the planimeter arm roughly at right angles with *A B* and press the knife edge lightly into the paper to mark the point of starting as at *X*. Carry the tracing point out over *A B* and around the diagram in the direction that a clock runs as indicated by the solid arrows and back over *A B* making another depression as at *Z* to mark the position of the knife edge when the tracing point is again at the center *A*. Then, being careful to move neither the tracing point nor the knife edge, revolve the diagram about 180° using the tracing point as a center, bringing it into the dotted position of Fig. 101. Having secured the diagram in this position trace it again in the opposite

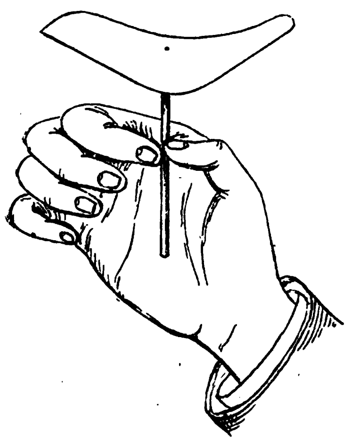


Fig. 100.

direction from that followed by the hands of a watch as shown by the dotted arrows, and make still another depression to mark the position of the knife edge when the tracing point returns to the center. This will probably be somewhere near *X* as at *Y*. We have now three marks *X*, that at which the knife edge started, *Z*, that to which it departed, and *Y*, that to which it returned when the diagram was retraced. For plainness I have reproduced them at the left. Make a mark as "*a b*" half way between *X Y*, then the distance between this mark and *Z*, *i. e.*, the length of the dotted line, multiplied by the length of the planimeter arm *A B*, Fig. 99 will be the area in square inches approximately, and the approximation will be very close when the arm is of considerable length compared with the area to be measured. By making the planimeter arm ten inches in length the multiplication may be done by shifting the decimal point, or as each inch of length will indicate ten square inches the area may be measured directly by taking the distance *Z X* with a scale of ten to the inch, each tenth representing ten square inches, or a scale of 100 to the inch each unit of which would represent one-tenth of a square inch.

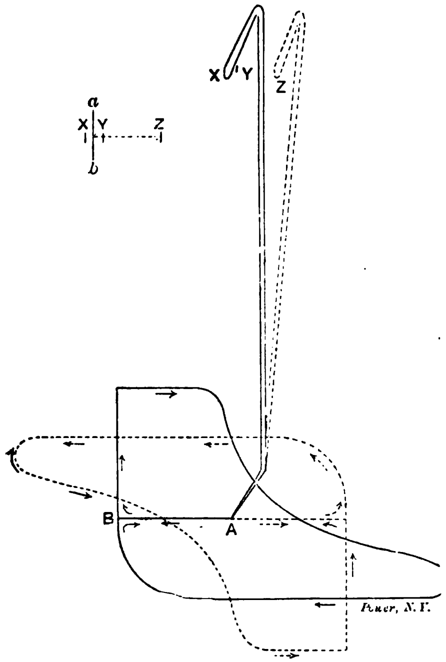


Fig. 101.

In Fig. 102, start with the tracing point at *A* and the wheel at zero and trace the rectangle *A B C D*. The wheel motion gained in moving from *A* to *B* is neutralized by the movement from *C* to *D*. The line *B C* is in the neutral axis, so the wheel gets no movement while the tracer passes over it. When the point arrives at *D*, therefore, the wheel will have returned to zero, and

the full area of the rectangle will be recorded while the tracing point passes down the line D A. For a rectangle, therefore, with its left hand edge in the neutral line of the instrument, all that is necessary to find the area is to start at the upper right-hand corner with the wheel at zero and carry the tracing point down the right-hand edge, as D A in Fig. 102. Conversely, if we have a given area recorded on the wheel, we can find the height of a rectangle of equal area for a given length by running the tracing point up the line marking its right-hand edge (the left being in the neutral line), until the wheel returns to zero. Suppose, for instance, we start at A, Fig. 102, with the planimeter wheel at zero and trace the outline of the indicator diagram. When the tracing point gets around to A again the area of the diagram will be recorded on the wheel. Now, suppose we run the tracing point up the line A D until the wheel comes back to zero, the line A D will be the average height of the indicator diagram, that is the height of a rectangle of equal area, and by measuring the length of A D with the scale corresponding to the spring with which the diagram was taken, we find the mean effective pressure of the diagram at once without calculation.

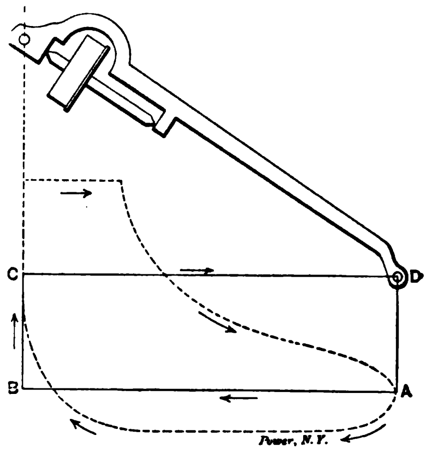


Fig. 102.

This principle is made use of in the Coffin Averaging Instrument, a form of planimeter especially adapted to measuring the mean effective pressure represented by indicator diagrams and shown in Fig. 103. The indicator card is placed under the clips A and C, with the atmospheric line parallel with the horizontal leg of the stationary clip C, and the left hand edge of the diagram against the inside vertical edge of that clip. The inside edge of the movable clip A is then placed against the right-hand extremity of the diagram, so that the length of the diagram is

just included between the two clips. The tracing point of the planimeter is then placed upon any portion of the diagram which is against the right-hand clip, the wheel set to zero and the point gently pressed into the paper to mark the starting point. The tracing point is then carried around the diagram in the direction of the hands of a watch, and when it returns to the point from which it started the area of the diagram will be recorded upon the wheel. No attention need be paid to this reading. Simply carry the point upward against the edge of the clip *A* until the wheel returns to zero, at which point press the tracer again into the paper. The distance between the starting point and the point thus made will be the average height of the diagram and measured with the scale of the spring with which the diagram was taken will give at once the mean effective pressure. It is not necessary even to set the wheel at zero in starting. You can record the reading, whatever it may be, after the tracer has been set at the starting point, trace the diagram and then run the tracing point upward beside the clip until the wheel returns to the reading with which you started. The whole apparatus is mounted on a rose-wood board with an inset tablet of suitable surface for the planimeter wheel to run upon. A weight *Q* is placed upon the end opposite the tracing point to hold it in the guiding groove.

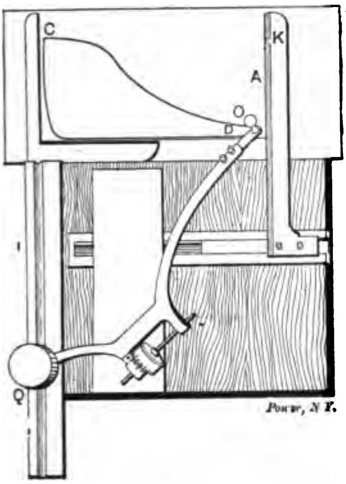


Fig. 103.

CHAPTER XIII.

COMPUTING THE HORSE POWER.

Force is that which tends to produce motion or change of motion in matter. The pressure of steam or of water under a head, the pull of a weight, the pull or push of a muscle, are all familiar examples of force.

When force is exerted through space, *Work* is done. The full steam pressure may stand upon the engine piston for hours, but no work will be done unless the piston moves. A suspended weight does no work except while it is being lowered, and it is only in its ability to be lowered, *i. e.*, in its elevated position, that its capacity for doing work exists.

The *Foot-Pound* is the unit of work or energy. It is the equivalent of one pound of force exerted through one foot of space. To lift one hundred pounds one foot would require 100 foot-pounds of energy, as it would also to lift one pound 100 feet. If a horse has to pull 50 pounds to draw a wagon, and draws it 100 feet, he will develop 5,000 foot-pounds. Notice that this has no reference to the weight of the wagon, simply to the force required to drag it.

A Horse-Power is the *unit of the rate of development*, or of consumption of energy or of work. It is 550 foot-pounds per second, 33,000 foot-pounds per minute, or 1,980,000 foot-pounds per hour.

It will be seen that the indicator gives us a means of determining the average force pushing the piston forward (the mean effective pressure per square inch multiplied by the number of square inches in the piston), and this multiplied by the number of feet through which the piston moves in a minute and divided by 33,000 will give the horse-power which the engine is developing.

The simplest formula for horse power is therefore,

$$\frac{\text{Area} \times \text{M. E. P.} \times \text{Piston Speed}}{33{,}000} = \text{H. P.}$$

The Area of the piston is found by multiplying the square of the diameter of the cylinder by .7854. Table X at the end of the volume renders this calculation unnecessary.

The mean effective pressure (M. E. P.) is found by measurement from the diagram, as explained in the previous chapter.

The piston speed is the number of feet through which the pressure acts upon the piston per minute. In a double acting engine, (that is, an engine which takes steam at each stroke, or twice a revolution) this is the revolutions per minute $\times$ 2 $\times$ the length of the stroke in feet. If the engine is single acting but takes steam every revolution the piston speed is the product of the revolutions per minute and the stroke in feet. Most gas engines make a working stroke once in two revolutions and their piston speed when this is done is the stroke in feet times one-half the revolutions per minute. Usually the governing of gas engines is accomplished by skipping a charge, in which case their piston speed can only be determined by counting the explosions, the piston speed being the stroke in feet multipled by the number of explosions per minute.

A simple and easily remembered formula for horse-power is

$$\frac{P\ A\ N\ S}{33,000} = \text{H. P.}$$

Where P = mean effective pressure,

A = area of piston in square inches,

N = number of working *strokes* per minute,

S = length of stroke in feet.

RULE:—*Multiply together the mean effectioe pressure, the area of the piston in square inches, the number of working strokes per minute, and the length of the stroke in feet and divide by 33,000. The quotient will be the horse-power.*

EXAMPLE:—What is the horse-power developed by a 24 x 48 inch engine running at 70 revolutions per minute with 42 pounds M. E. P. ?

The pressure	P = 42	lbs.	given
" area	A = 452.39	sq in	24^2 x .7854
" number	N = 140	stroke per min.	70 x 2
" stroke	S = 4	feet	given

$$\frac{P\ A\ N\ S}{33,000} \quad \frac{42 \times 452.39 \times 140 \times 4}{33,000} = 322.43$$

In figuring out a number of diagrams from one engine running at a constant speed it is most convenient to compute first the horse-power developed per pound of mean effective pressure, and multiply this "horse-power constant" by the mean effective pressure of each diagram to find the horse-power represented by that diagram. This can be done by considering the M. E. P. in formula 1 as unity, in which case, as it is a multiplier, it may be left out, and we get

$$\frac{\text{Area} \times \text{Piston Speed}}{33{,}000} \text{ or } \frac{A\ N\ S}{33{,}000} = \text{H.P. per pound of M.E.P.}$$

and this H. P. constant multiplied by M. E. P. = H. P.

TO FIND THE HORSE-POWER CONSTANT OR HORSE-POWER PER POUND OF MEAN EFFCTIVE PRESSURE,

RULE: *Multiply the piston area in square inches by the piston speed in feet per minute and divide by 33,000*, or

Multiply together the piston area in square inches, the number of working strokes per minute, and the stroke in feet, and divide the product by 33,000.

EXAMPLE: What is the horse-power constant of the above engine?

$$\frac{A\ N\ S}{33{,}000} = \frac{452.39 \times 140 \times 4}{33{,}000} = 7.6769$$

This multiplied by the mean effective pressure will give us the horse-power thus

$$7.6769 \times 42 \quad 322.4298$$

as before.

As an assistance in this work the author prepared some years since Table II which gives the horse-power constants for various diameters at any piston speed. Opposite the diameters given in the left hand column will be found the horse-power which would be developed at 1, 2, 3, 4, 5, 6, 7, 8, and 9 feet of piston speed per minute. If it is desired to know what power a 24-inch engine would develop at 500 feet of piston speed, look opposite 24 and under 5, and find .06854 as the horse-power developed at 5 feet piston speed per minute. At 500 feet the horse-power would be 100 times as much, or 6.8544 horse-power per pound of mean effective pressure, the multiplication by 100 being easily done by shifting the decimal point two places to the right. Suppose it is desired to find the horse-power constant for

Diameter in Inches.	1	2	3	4	5	6	7	8	9
1	.00002	.00004	.00007	.00010	.00012	.00014	.00017	.00020	.00021
4	.00038	.00077	.00114	.00152	.00190	.00228	.00267	.00305	.00343
4¼	.00042	.00085	.00129	.00172	.00215	.00258	.00301	.00344	.00387
4½	.00045	.00090	.00136	.00181	.00226	.00271	.00316	.00361	.00406
4¾	.00054	.00107	.00162	.00215	.00269	.00322	.00376	.00430	.00483
5	.00060	.00119	.00179	.00238	.00298	.00357	.00417	.00476	.00536
5¼	.00066	.00131	.00197	.00262	.00328	.00394	.00459	.00525	.00590
5½	.00072	.00144	.00216	.00288	.00360	.00432	.00504	.00576	.00648
5¾	.00079	.00157	.00236	.00315	.00393	.00472	.00551	.00630	.00708
6	.00086	.00171	.00257	.00343	.00428	.00514	.00600	.00685	.00771
6¼	.00093	.00186	.00279	.00372	.00465	.00558	.00651	.00744	.00837
6½	.00100	.00201	.00302	.00402	.00503	.00603	.00704	.00804	.00905
6¾	.00108	.00217	.00325	.00434	.00542	.00651	.00759	.00868	.00976
7	.00117	.00233	.00350	.00467	.00583	.00700	.00816	.00933	.01050
7¼	.00125	.00250	.00375	.00500	.00626	.00750	.00876	.01001	.01126
7½	.00134	.00268	.00402	.00535	.00669	.00803	.00937	.01071	.01205
7¾	.00143	.00286	.00429	.00572	.00715	.00858	.01001	.01144	.01287
8	.00152	.00305	.00457	.00609	.00762	.00914	.01066	.01219	.01371
8½	.00172	.00344	.00516	.00688	.00860	.01032	.01204	.01376	.01548
9	.00193	.00386	.00578	.00771	.00964	.01157	.01349	.01542	.01735
9½	.00215	.00430	.00644	.00859	.01074	.01289	.01504	.01718	.01923
10	.00238	.00476	.00714	.00952	.01190	.01428	.01666	.01904	.02142
10½	.00262	.00525	.00787	.01050	.01312	.01574	.01837	.02099	.02362
11	.00288	.00576	.00864	.01152	.01440	.01728	.02016	.02304	.02592
11½	.00315	.00630	.00944	.01259	.01574	.01889	.02203	.02518	.02833
12	.00343	.00685	.01028	.01371	.01714	.02056	.02399	.02742	.03084
13	.00402	.00805	.01207	.01609	.02011	.02413	.02816	.03218	.03620
14	.00466	.00933	.01399	.01866	.02332	.02799	.03265	.03732	.04198
15	.00536	.01071	.01607	.02142	.02678	.03213	.03749	.04284	.04820
16	.00609	.01219	.01828	.02437	.03046	.03656	.04265	.04874	.05484
17	.00688	.01376	.02063	.02751	.03439	.04127	.04815	.05503	.06190
18	.00771	.01542	.02313	.03084	.03856	.04627	.05398	.06169	.06940
19	.00859	.01718	.02578	.03437	.04296	.05155	.06014	.06873	.07733
20	.00952	.01904	.02856	.03808	.04760	.05712	.06664	.07616	.08568
21	.01049	.02099	.03149	.04198	.05248	.06297	.07347	.08397	.09446
22	.01152	.02304	.03456	.04608	.05760	.06912	.08063	.09215	.10367
23	.01259	.02518	.03777	.05036	.06295	.07554	.08813	.10072	.11331
24	.01371	.02742	.04113	.05484	.06854	.08225	.09596	.10967	.12338
25	.01488	.02975	.04463	.05950	.07438	.08925	.10413	.11900	.13388
26	.01609	.03218	.04827	.06436	.08044	.09653	.11262	.12871	.14480
27	.01735	.03470	.05205	.06940	.08675	.10410	.12145	.13880	.15615
28	.01866	.03732	.05598	.07464	.09330	.11196	.13061	.14927	.16793
29	.02002	.04003	.06005	.08006	.10008	.12009	.14011	.160 3	.18014
30	.02142	.04284	.06426	.08568	.10710	.12852	.14994	.17136	.19278
31	.02287	.04574	.06862	.09149	.11436	.13723	.16010	.18297	.20585
32	.02437	04874	.07311	.09748	.12186	.14623	.17060	.19497	.21934
33	.02592	.05184	.07775	.10367	.12959	.15551	.18143	.20735	.23326
34	.02751	.05503	.08254	.11005	.13756	.16508	.19259	.22010	.24761
35	.02915	.05831	.08746	.11662	.14577	.17493	.20408	.23324	.26239
36	.03084	.06169	.09253	.12338	.15422	.18507	.21591	.24676	.27760
37	.03258	.06516	.09775	.13033	.16291	.19549	.22807	.26066	.29324
38	.03437	.06873	.10310	.13747	.17183	.20620	.24057	.27494	.30930
39	.03620	.07240	.10860	.14480	.18100	.21720	.25340	.28960	.32580
40	.03808	.07616	.11424	.15232	.19040	.22848	.26656	.30464	.34272
41	.04001	.08002	.12002	.16003	.20003	.240 5	.28005	.32006	.36007
42	.04198	.08397	.12595	.16793	.20992	.25190	.29388	.33586	.37785
43	.04401	.08801	.13202	.17602	.22003	.26404	.30804	.35205	.39605
44	.04608	.09215	.13823	.18431	.23038	.27646	.32254	.36861	.41469
45	.04819	.09639	.14458	.19278	.24097	.28917	.33736	.38556	.43375
46	.05036	.10072	.15108	.20144	.25180	.30216	.35252	.40288	.45325
47	.05257	.10515	.15772	.21030	.26287	.31544	.36802	.42059	.47317
48	.05484	.10967	.16451	.21934	.27418	.32901	.38385	.43868	.49352
49	.05714	.11429	.17143	.22857	.28572	.34286	.40001	.45715	.51429
50	.05950	.11900	.17850	.23800	.29750	.35700	.41650	.47600	.53550
51	.06190	.12381	.18571	.24761	.30952	.37142	.43333	.49523	.557 3
52	.06436	.12871	.19307	.25742	.32178	.38613	.45049	.51484	.57920
53	.06685	.13371	.20056	.26742	.33427	.40112	.46798	.53483	.60 69
54	.06940	.13880	.20820	.27760	.34700	.41640	.48580	.55520	.62461
55	.07199	.14399	.21598	.28798	.35997	.43197	.50396	.57595	.64795
56	.07464	.14927	.22391	.29855	.37318	.44782	.52246	.59707	.67173
57	.07732	.15465	.23198	.30930	.38663	.46396	.54128	.61861	.69593
58	.08006	.16013	.24019	.32026	.40032	.48038	.56044	.64050	.72057
59	.08285	.16570	.24854	.33139	.41424	.49709	.57993	.66278	.74563
60	.08568	.17136	.25704	.34272	.42840	.51408	.59976	68544	.77112

a cylinder 30 inches in diameter at 532 feet of piston speed. The constant

At 500	feet	is	10.76
" 30	"	"	.6426
" 2	"	"	.04284
" 532	"	"	11.44544

The constant at 30 feet, as will readily be seen, is found at a glance by multiplying the constant given in the table for 3 feet by 10, by moving the decimal point one place to the right. In the same way the constant for fractional parts of a foot can be found by shifting the decimal to the left; the constant of a 30-inch cylinder for .3 of a foot being .006426 horse-power. For example, suppose we have a 32 by 64 inch engine, running at 62 revolutions per minute. The piston speed is

$$\frac{64 \times 2 \times 62}{12} = 661.3 \text{ feet per minute}$$

Constant for 600.0 ft.	=	14.623
60.0 "	=	1.4623
1.0 "	=	.02437
.3 "	=	.007311
		16.116981

Of course it is not necessary to use a constant with so many decimal places in figuring the diagrams, and they may be discarded, retaining only so many as are necessary to the degree of accuracy required by the case in hand, increasing the last figure retained by one if the left hand figure of the portion discarded is greater than 5. For instance, if we wished to use only two places of decimals in the above example we should call the constant 16.12, the first figure of the 6981 discarded being greater than 5.

Having found in this way the horse-power developed per pound of mean effective pressure, we can easily find the horse-power represented by any diagram from the given engine, by multiplying the horse-power constant so found by the mean effective pressure shown by the diagram. An engineer, for instance, having once worked out the constant for his engine needs only to figure the mean effective of his diagrams each day and multiply them by the constant to find the horse-power. On a

test the average M. E. P. of a series of diagrams multiplied by the horse-power constant will give the average horse-power.

When the mean effective pressure used with this formula is that measured from a diagram from one end of the cylinder it must be understood that the result will be the average horse-power of a double acting engine only when the diagram from the other end has precisely the same area. To obtain the average horse-power of a double acting engine, *i. e.*, an engine which uses steam in both ends of the cylidner, as is most commonly the case, we must find either *a*, the average mean effective pressure of the two ends and use this as the mean effective pressure ***P*** of the formula or *b*, the horse-power developed by each end and add them together. The first method is the more simple. You measure the mean effective pressure of the diagram from each end, add them together, divide by 2 and use the result as the ***P*** of the formula or the mean effective pressure of the rule.

Allowance for the piston rod must be made when accuracy is desired. Each pound of mean effective pressure will do less work in the crank end than in the head end because it has less area to work upon, owing to the fact that some of the area of the piston is covered by the piston rod. If you wish to allow for this you must, when you work with the average mean effective pressure, make the area, the *A* of the formula, less than the area of the piston by one-half the area of the piston rod.

HORSE POWER CORRECTED FOR PISTON ROD.

$$\frac{P\left(A-\frac{a}{2}\right)NS}{33{,}000} = HP$$

where $a =$ the cross sectional area of the piston rod and $P =$ the average M. E. P. of the diagrams from both ends.

RULE: *Subtract from the area of the piston one-half the cross sectional area of the rod. Multiply together the remainder, the average mean effective pressure of the diagrams from both ends, the number of working strokes per minute and the stroke in feet, and divide by 33,000. The quotient will be the horse-power.*

EXAMPLE: Suppose with the above engine 24 x 48 at 70 revolutions per minute the M. E. P. of the diagram from the head end is 39, of that from the crank end 45, and that it had a 4-inch rod. The average M. E. P. would be

$$\frac{39+45}{2} = 42 \text{ pounds}$$

and the average area in use would be

$$452.39 - \frac{12.466}{2} = 446.157$$

12.466 being the area of a 4-inch circle. The average horse-power of both ends corrected for the piston rod would be

$$\frac{42 \times 446.157 \times 140 \times 4}{33,000} = 317.99$$

If for any reason it is desired to know how much horse-power is developed during each separate stroke, the area of each end must be considered separately and each end credited with only its own strokes. The head end has the full diameter of the piston, which for the crank end is diminished by the full area of the rod, and the number of strokes is equal for each end only to the number of revolutions instead of being twice that number as when the engine is considered as a whole. In the above example we would have

FOR THE POWER OF THE HEAD END

$$\frac{39 \times 452.39 \times 70 \times 4}{33,000} = 149.70 \; H.\ P.$$

and

FOR THE POWER OF THE CRANK-END

$$\frac{45 \times (452.39 - 12.466) \times 70 \times 4}{33,000} = 167.97 H.\ P.$$

The sum of these will be the power of the engine as a whole and is as we found it above. $149.70 + 167.97 = 317.67$ *H.P.* the fractional difference coming from dropping decimals.

In strictness, in order to find the power which is being developed by one end of the cylinder we should use a diagram made of the line showing the forward pressure in the end which we are computing, and the back pressure or counter pressure line of the diagram from the other end. The counter pressure line diagram from the head end does not show the back pressure against the piston when the head end was doing work but while the piston is being forced backward by the steam in the crank-end. The effective pressure at any time in the forward stroke is the pressure in the head-end at that instant minus the pressure in the crank-end, and to get the proper mean effective pressure during the forward stroke we should take the mean pressure on the head-end less the mean back pressure on the crank-end. This would make no difference in the computed

power of the engine as a whole for what was lost on one end would be gained by the other, but it would, if the back pressure lines were different, affect the amounts of power indicated at the different ends, and comes into the question of balancing the load equally. In New England factories it is common to run an engine one-half condensing, that is, to have a separate exhaust pipe for each end, one running to the condenser and the other end exhausting, perhaps under a back pressure for heating, etc. The diagrams from such an engine would be like Fig. 104 obviously the load would not be equally divided between the two ends of the cylinder when the areas of the diagrams were equal.

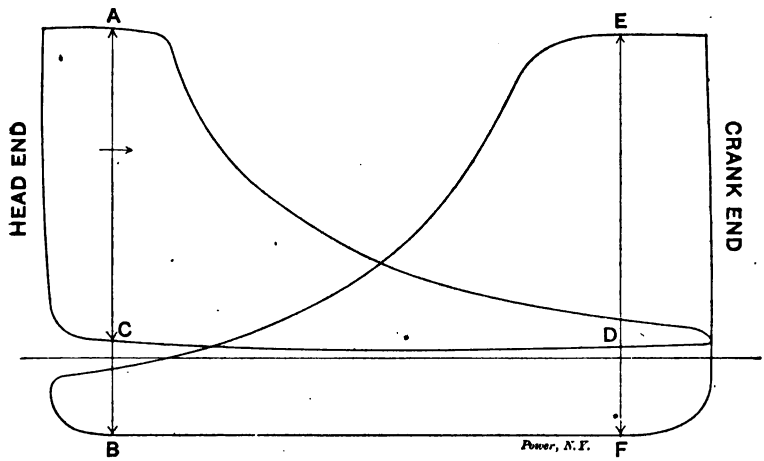


Fig. 104.

When the piston is on the line *A B* and moving in the direction of the arrow there is a pressure urging it forward proportional to the height of *A* and the back pressure is proportional only to the heigth of *B* so that the effective pressure is *A B*, although if we take the back pressure line of the head-end diagram it will appear to be only *A C*. The diagram from the crank-end would appear, taken by itself, to have an effective pressure proportional to *E F* when the piston was at that point in the stroke, but since the piston is moving against a back pressure proportional to the height of *D* the effective pressure at that point is *D E*. The effort of both ends upon the crank pin cannot be balanced by making the area of the crank-end diagram equal to that of the

head-end. The work actually done upon the crank when the piston is moving forward is found by combining the back pres-

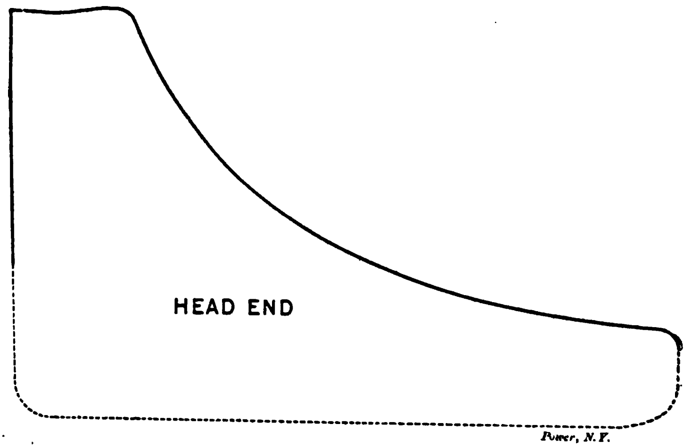


Fig. 105.

sure line of the crank-end diagram with the forward pressure line of the head-end diagram as in Fig. 105 and vice versa for the backward strokes as in Fig. 106. The work will be equalized between the two ends when the area of these reconstructed

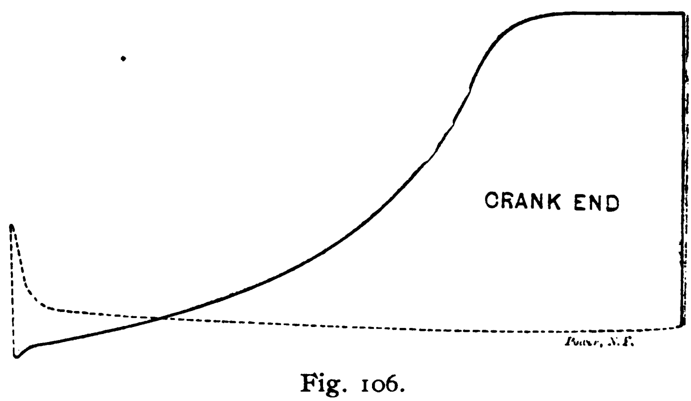


Fig. 106.

diagrams are equal, proper allowance being made for the piston rod.

CHAPTER XIV.

MEAN EFFECTIVE PRESSURE AND POINT OF CUT-OFF BY COMPUTATION.

It is often necessary to calculate the mean effective pressure which will result from using steam of a given pressure with a certain number of expansions. This may be done by finding the mean effective pressure which would be produced under ideal conditions and then making a reduction for the departures from those conditions unavoidable in practice.

We have seen in chapter VIII that the mean effective pressure is proportional to the area of the diagram. If a diagram, whatever its shape, has twice as much area as another diagram of the same scale and length, it will represent twice the mean effective pressure. If the initial pressure were continued throughout the stroke the diagram would ideally be a rectangle like *a b c d*, Fig. 107. If the cut-off occurred at *e* and there were no clearance and no back pressure above absolute zero, the diagram would ideally follow the lines *a e f c d* and the mean effective would be to the initial pressure as the area *a e f c d* is to the area *a b c d*. Now since the hyperbola bounding one part of the diagram is fixed for any initial pressure and ratio of expansion, it follows that the area *e f c g* beneath the expansion line bears a fixed ratio to the rectangle *a e g d* representing the volume of steam involved at

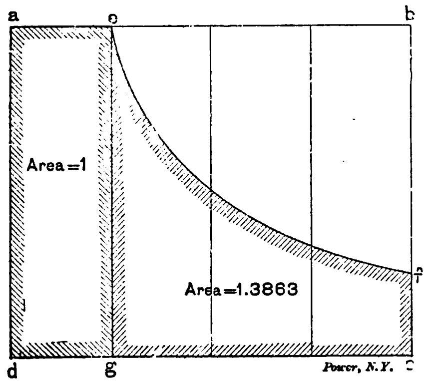


Fig. 107.

initial pressure, for any given ratio of expansion. For instance, with a ratio of expansion of 4, i.e., when the final volume is four times the initial volume, as in Fig. 107, the area *e f c g* will be 1.3863 times as large as the area *a e g d*, and it will be so whatever the size and scale of the diagram so long as *e f* is a true hyperbola and the distance *d c* is four times as great as the distance *a e*. If we call the area *a e g d* unity, then the area of the large rectangle *a b c d* would be represented by the ratio of expansion = 4; the area of the diagram *a e f c g d* would be 1 + 1.3863 = 2.3863, and the initial pressure would be to the mean effective pressure as 4 is to 2.3863.

Or, looking at it in another way, the mean effective pressure is the area divided by the length. In Fig. 107 the area is 2.3863 and the length *d c* is 4 since *d g* = 1. Then the mean effective pressure per pound of initial pressure is $\frac{2.3863}{4}$

The ratio between the areas before and after the point of cut-off is simply the hyperbolic logarithm of the ratio of expansion. In the above example 1.3863 is the hyperbolic logarithm of 4, and by consulting a table of hyperbolic logarithms (see Table IV page 122) we can find the value for any ratio of expansion just as simply as we can find the area of a circle of given diameter in a table of areas. Then as the ratio of expansion is to 1 plus the hyperbolic logarithm of that number, so is the initial to the mean pressure.

Ratio : 1 + hyp. log. ratio :: Initial : mean, or

$$\frac{1 + \text{hyp. log. ratio} \times \text{Initial}}{\text{Ratio expansion}} = \text{mean pressure}$$

RULE:—*Add 1 to the hyperbolic logarithm of the ratio of expansion; multiply the sum by the absolute initial pressure, and divide by the ratio of expansion. The quotient will be the mean forward absolute pressure represented by the ideal diagram.*

EXAMPLE:—What will be the ideal mean forward absolute pressure with steam of 100 pounds gage pressure, cut off at ¼ stroke? The absolute initial pressure will be 114.7 pounds, the ratio of expansion 4, the hyperbolic logarithm of 4 = 1.3863, then

$$\frac{1 + 1.3863 \times 114.7}{4} = 68.427 \text{ pounds}$$

This would be the mean effective pressure if the full initial pressure were promptly realized in the cylinder and maintained up to cut-off, if the expansion line were a perfect hyperbola carried to the extreme end of the stroke, and if the backward stroke were commenced and completed with a perfect vacuum before the piston, —in short, if the diagram followed the lines *a e f c d*, Fig. 108. As a matter of fact we must count upon a loss between the boiler and initial pressures; we cannot realize a a perfect vacuum in the cylinder, and there will be a rounding-off of the corners, so that even under favorable circumstances the diagram would be more like the shaded outlines inclosed within the ideal. The loss of pressure represented between the ideal and the realized forward pressure lines in Fig. 108 is a little over three pounds; that between the line of zero pressure and the counter-pressure line of the condensing diagram, which represents a realized vacuum of 12 pounds or 26 inches, 3.5 pounds; and that between the zero line and the counter-pressure line of the non-condensing diagram (which represents a back pressure on the straight portion of the line of one pound above the atmosphere) 16.5 pounds. The loss on the forward stroke is mostly due to the loss in the initial pressure, which has been assumed to be 5 pounds, the ideal diagram being drawn with an initial pressure of 100 pounds, and one-quarter cut off. At this cut-off each pound lost in the initial pressure decreases the mean effective pressure .596 of a pound. The loss of 5 pounds would reduce it $5 \times .596 = 2.98$. The further loss due to the rounding off of the corners at cut-off and release increases this to something over three pounds. If we call the loss on the forward stroke 3.5 pounds we could find the mean effective of the condensing diagram by subtracting from the 68.43 pounds represented by the ideal diagram $3.5 + 3.5 = 7$ pounds, and that represented by the non-condensing diagram by subtracting from 68.43, $16.5 + 3.5 = 20$ pounds.

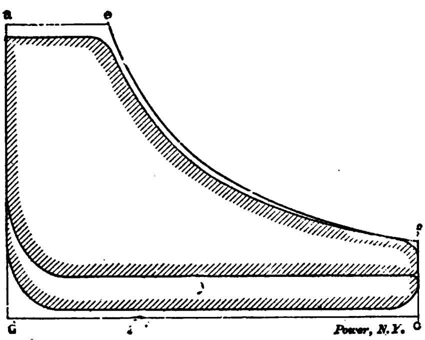


Fig. 108.

In Table III are given the mean effective pressures which would be produced by the initial pressures at the left at points of cut-off from $\frac{1}{10}$ to ¾ *under the conditions of the ideal diagram.* Subtract from the values here given 7 pounds for condensing engines and 20 pounds for non-condensing engines and you will have the mean effective which will be realized in the cylinder with diagrams as good as those shown in Fig. 108. For other conditions the losses from wire-drawing and back pressure can be estimated and the reduction made by the use of Table IV. Opposite each ratio in this table is given the variation in mean effective pressure for each pound of initial, so that the loss for any falling-off in realized pressure may be computed, or the mean effective for any initial not given in the table may be found by multiplying the given absolute initial by the M. E. P. per pound of initial absolute as given in the table.

The various quantities calculated in Table IV are obtained by variations of the original proportion.

Ratio: 1 + hyp. log. of Ratio :: Initial : Mean Pressure.

The different transpositions are to be found in the headings of the columns. The various methods in which this table may be used are as follows:

TO FIND THE MEAN PRESSURE.

Opposite the given point of cut-off or ratio of expansion find in the fifth column the mean pressure per pound of initial. Multiply this value by the given absolute initial pressure. The result will be the mean pressure sought.

Example : — What would be the mean pressure with 80 pounds initial, cutting off at one-fifth stroke?

In the column of points of cut-off find one-fifth or .2. Opposite in the column of mean pressure per pound of initial is .52188. Our initial pressure is 80 pounds gage, say 95 absolute, and

$$95 \times .52188 = 49.5786 \text{ lbs.}$$

TO FIND THE POINT OF CUT-OFF OR RATIO OF EXPANSION.

Divide the absolute initial pressure by the desired mean pressure. In the column of initial per pound of mean pressure find the number most nearly corresponding to the quotient, and the point of cut-off and ratio of expansion will be found in the same horizontal line.

TABLE III.

Cut off at		$\frac{1}{10}$=.1	$\frac{2}{19}$=.10	$\frac{1}{9}$=.111	$\frac{2}{17}$=.11	$\frac{1}{8}$=.125	$\frac{2}{15}$=.133	$\frac{1}{7}$=.143	$\frac{2}{13}$=.151	$\frac{1}{6}$=.167	$\frac{2}{11}$=.18	=.[illegible]	$\frac{2}{9}$=.22	$\frac{1}{4}$=.25	$\frac{3}{10}$=.3	$\frac{1}{3}$=.333	$\frac{3}{8}$=.375	$\frac{2}{5}$=.[illegible]	$\frac{1}{2}$=.5	$\frac{3}{5}$=.6	$\frac{5}{8}$=.625	$\frac{2}{3}$=.667	$\frac{7}{10}$=.7	$\frac{3}{4}$=.75
Apparent ratio of Expansion		10	9.5	9	8.5	8	7.5	7	6.5	6	5.5	5	4.5	4	3.33	3	2.67	2.5	2	1.67	1.6	1.5	1.43	1.33
M. E. P. per lb. Initial		.330	.342	.355	.369	.385	.402	.421	.442	.465	.492	.523	.556	.596	.661	.699	.743	.773	.816	.906	.919	.987	.949	.964
Initial Pressure. Gauge.	Absolute.	MEAN EFFECTIVE PRESSURE FROM FULL AREA OF IDEAL DIAGRAM.																						
25	39.7	13.11	13.58	14.09	14.65	15.28	15.96	16.71	17.55	18.46	19.53	20.72	22.07	23.68	26.24	27.77	29.49	30.43	33.61	35.96	36.47	37.20	37.69	38.2[illegible]
30	44.7	14.76	15.28	15.87	16.49	17.21	17.97	18.82	19.76	20.79	21.99	23.33	24.85	26.67	29.54	31.27	33.20	34.26	37.84	40.49	41.07	41.88	42.44	43.0[illegible]
35	49.7	16.41	17.00	17.64	18.34	19.13	19.98	20.92	21.97	23.11	24.45	25.94	27.63	29.65	32.85	34.77	36.91	38.10	42.07	45.02	45.66	46.57	47.19	47.91
40	54.7	18.07	18.71	19.42	20.18	21.06	21.99	23.03	24.18	25.44	26.91	28.55	30.41	32.63	36.15	38.26	40.63	41.93	46.31	49.55	50.26	51.25	51.91	52.73
45	59.7	19.72	20.42	21.19	22.03	22.98	24.	25.12	26.39	27.76	29.37	31.16	33.19	35.62	39.46	41.76	44.34	45.76	50.54	54.08	54.85	55.94	56.68	57.5[illegible]
50	64.7	21.37	22.13	22.97	23.87	24.91	26.01	27.23	28.60	30.09	31.83	33.77	35.97	38.60	42.76	45.26	48.06	49.59	54.77	58.6[illegible]	59.44	60.62	61.43	62.[illegible]
55	69.7	23.02	23.84	24.74	25.72	26.83	28.02	29.34	30.81	32.31	34.29	36.38	38.75	41.58	46.07	48.76	51.77	53.43	59.00	63.15	64.05	65.31	66.18	67.19
60	74.7	24.67	25.55	26.52	27.56	28.75	30.03	31.45	33.02	34.74	36.75	38.99	41.53	41.56	49.37	52.26	55.48	57.26	63.24	67.67	68.63	69.99	70.92	72.01
65	79.7	26.32	27.26	28.29	29.41	30.68	32.04	33.55	35.23	37.06	39.21	41.59	44.31	47.55	52.67	55.75	59.20	61.09	67.47	72.20	73.22	74.68	75.67	76.83
70	84.7	27.97	28.97	30.07	31.25	32.60	34.05	35.66	37.44	39.39	41.67	44.20	47.09	50.53	55.98	59.25	62.91	64.92	71.70	76.73	77.82	79.36	80.42	81.64
75	89.7	29.62	30.68	31.84	33.10	31.53	36.06	37.76	39.65	41.71	44.13	46.81	49.87	53.51	59.28	62.75	66.62	68.76	75.94	81.26	82.41	84.05	85.17	86.27
80	94.7	31.28	32.39	33.62	34.91	36.45	38.07	39.87	41.86	41.04	46.59	49.42	52.65	56.50	62.59	66.25	70.34	72.59	80.17	85.79	87.01	88.73	89.91	91.28
85	99.7	32.93	34.10	35.39	36.19	38.38	40.08	41.97	44.07	46.36	49.05	52.03	55.43	59.48	65.89	69.74	74.05	76.42	84.40	90.32	91.60	93.42	94.66	96.10
90	104.7	34.58	35.81	37.17	38.63	40.30	42.09	44.08	46.28	48.69	51.51	54.64	58.21	62.46	69.20	73.24	77.76	80.26	88.63	91.84	96.19	98.10	99.41	100.92
95	109.7	36.23	37.52	38.94	40.48	42.23	44.10	46.18	48.49	51.01	53.97	57.25	60.99	65.44	72.50	76.74	81.48	84.09	92.87	99.37	100.79	102.79	104.15	105.74
100	114.7	37.88	37.78	40.72	42.32	44.15	46.11	48.20	50.70	53.34	56.43	59.86	63.77	68.43	75.81	80.24	85.19	87.92	97.10	103.90	105.38	107.47	108.90	110.56
110	124.7	41.18	42.65	44.27	46.01	48.00	50.13	52.50	55.12	57.98	60.35	65.08	69.33	71.39	82.41	87.23	92.62	95.[illegible]9	105.26	112.96	114.57	116.84	118.39	120.20
120	134.7	44.49	46.07	47.82	49.80	51.85	54.15	56.71	59.54	62.64	66.27	70.30	71.89	80.36	89.02	94.23	100.05	103.2[illegible]	114.04	122.02	128.78	126.21	127.89	129.84
130	144.7	47.79	49.49	51.37	53.19	55.70	58.17	60.92	63.96	67.29	70.19	75.52	80.45	86.32	95.63	101.22	107.45	110.91	122.50	131.08	132.94	135.58	137.38	139.48
140	154.7	51.09	52.91	54.92	57.08	59.55	62.19	65.13	68.38	71.94	76.11	80.74	86.01	92.29	102.24	108.22	114.90	118.58	130.96	140.14	142.13	144.95	146.88	149.12
150	164.7	54.39	56.33	58.47	60.77	63.40	66.21	69.34	72.80	76.59	81.03	85.96	91.57	98.26	108.85	115.21	122.33	126.26	139.43	149.20	151.32	154.32	156.37	158.76
160	174.7	57.70	59.75	61.02	61.46	67.25	70.23	73.55	77.21	81.24	85.95	91.17	97.13	104.22	115.46	122.21	129.76	133.91	147.89	158.25	160.51	168.69	165.87	168.40
170	184.7	61.00	63.17	65.57	68.15	71.10	74.25	77.76	81.64	85.89	90.82	96.39	102.69	110.19	122.07	129.20	137.18	141.90	156.36	167.31	169.69	173.06	175.37	178.04
180	194.7	64.30	66.59	69.12	71.84	74.95	78.27	81.97	86.06	90.54	95.79	101.61	108.25	116.17	128.68	136.20	144.61	149.24	164.82	176.37	178.88	182.43	184.86	187.67
190	204.7	67.60	70.01	72.67	75.53	78.79	82.29	86.18	90.48	95.19	100.71	106.83	113.81	122.12	135.29	143.19	152.04	156.91	173.29	185.43	188.07	191.80	194.35	197.32
200	214.7	70.91	73.43	76.22	79.22	82.64	86.31	90.39	94.90	99.84	105.63	112.05	119.37	128.08	141.90	150.19	159.46	164.57	181.85	194.94	198.17	200.71	203.85	206.95
210	224.7	74.21	76.85	79.78	82.91	86.49	90.33	94.60	99.32	104.49	110.55	117.27	124.93	134.05	148.51	157.19	166.89	172.24	190.22	203.55	209.31	210.54	213.34	216.59
220	234.7	77.51	80.27	83.32	86.60	90.34	94.35	98.81	103.74	109.14	115.47	122.48	130.49	140.02	155.12	164.18	174.32	179.90	198.68	212.61	215.63	219.91	222.83	226.23
230	244.7	80.81	83.09	86.87	91.29	91.19	98.37	103.02	108.16	113.79	120.39	127.70	136.05	145.98	161.72	171.18	181.75	187.57	207.15	221.66	224.82	229.28	232.33	235.87
240	254.7	84.12	87.11	90.42	93.98	98.04	102.39	107.23	112.58	118.44	125.31	132.92	141.61	151.95	168.33	178.17	189.18	195.23	215.62	230.72	234.00	238.65	241.82	245.51
250	264.7	87.42	90.[illegible]3	93.97	97.67	101.89	106.11	111.41	117.00	122.09	130.23	138.14	147.17	157.91	174.94	185.17	196.60	202.90	224.08	239.78	243.19	248.02	251.32	255.15

To find the M. E. P. of a condensing diagram equivalent to Fig. 108. subtract 7 pounds from the ideal value found from the table: of a non-condensing diagram, 20 pounds.
The M. E. P. for any initial pressure not given in the table can be found by multiplying the (absolute) given pressure by the M. E. P. per pound of initial: as given in the third horizontal line of the table.

TABLE IV.

TABLE FOR COMPUTING MEAN AND INITIAL PRESSURES, POINTS OF CUT-OFF AND RATIOS OF EXPANSION.

Ratio.	Point of cut-off.	Hyperbolic Logarithm.	Initial per lb. mean pressure. $\frac{\text{Ratio}}{1+\text{hyp. log.}}=\frac{I}{M}$	Mean pressure per lb. initial. $\frac{1+\text{hyp. log.}}{\text{Ratio}}=\frac{M}{I}$	Ratio	Point of Cut-off.	Hyperbolic Logarithm.	Initial per lb. mean pressure. $\frac{\text{Ratio}}{1+\text{Hyp. log.}}=\frac{I}{M}$	Mean pressure per lb. initial. $\frac{1+\text{hyp. log.}}{\text{Ratio}}=\frac{M}{I}$
1.	1.00	0000	1.0000	1.00000	11.	.091	2.3978	3.22741	.30889
.1	.909	.0953	1.0043	.99572	.1	.090	2.4069	3.25809	.30693
.2	.833	.1823	1.0150	.98525	.2	.089	2.4159	3.27878	.30499
.3	.769	.2624	1.0298	.97.07	.3	.088	2.4248	3.29945	.30308
.4	.714	.3365	1.0475	.95464	.4	.088	2.4336	3.320 2	.30119
.5	.667	.4055	1.0372	.93700	.5	.087	2.4423	3.34079	.29933
.6	.625	.4700	1.0885	.91855	.6	.086	2.4509	3.36144	.29749
.7	.588	.5306	1.1107	.90035	.7	.085	2.4595	3.38211	.29568
.8	.556	.5878	1.1376	.88213	.8	.085	2.4681	3.40254	.29391
.9	.526	.6419	1.1572	.86416	.9	.084	2.4765	3.42293	.29214
2.	.500	.6931	1.1813	.84655	12.	.083	2.4349	3.44343	.29041
.1	.476	.7419	1.2056	.82930	.1	.083	2.4932	3.46387	.28869
.2	.456	.7885	1.2301	.81295	.2	.082	2.5014	3.48432	.28700
.3	.435	.8329	1.2548	.79691	.3	.081	2.5096	3.50477	.28533
.4	.417	.8755	1.2797	.78145	.4	.081	2.5176	3.52518	.28368
.5	.400	.9163	1.3046	.76652	.5	.080	2.5256	3.54549	.28205
.6	.385	.9555	1.3296	.75211	.6	.079	2.5336	3.56577	.28044
.7	.370	.9933	1.3545	.73826	.7	.078	2.5416	3.58599	.27882
.8	.357	1.0295	1.3795	.72486	.8	.078	2.5494	3.60621	.27722
.9	.345	1.0647	1.4046	.71199	.9	.078	2.5572	3.62645	.27572
3.	.333	1.0986	1.4295	.69953	13.	.077	2.5649	3.64606	.27422
.1	.323	1.1314	1 4544	.68751	.1	.076	2.5726	3.66679	.27272
.2	.313	1.1632	1.4793	.67600	.2	.075	2.5802	3.68694	.27123
.3	.303	1.1939	1.5042	.66482	.3	.075	2.5877	3.70711	.26975
.4	.294	1.2238	1.5289	.65406	.4	.074	2.5952	3.72719	.26827
.5	.286	1.2528	1.5536	.64366	.5	.074	2.6027	3.74724	.26681
.6	.278	1.2809	1.5783	.63358	.6	.073	2.6100	3.76731	.26543
.7	.270	1.3083	1 6029	.62387	.7	.073	2.6173	3.78736	.26404
.8	.263	1.3350	1.6274	.61447	.8	.072	2.6246	3.80738	.26265
.9	.256	1.3610	1.6518	.60528	.9	.072	2.6318	3.82731	.26128
4.	.250	1.3863	1.6762	.59658	14.	.071	2 6390	3.84728	.25993
.1	.244	1.4110	1.7005	.58805	.1	.071	2.6461	3.86725	.25859
.2	.238	1.4351	1.7248	.57976	.2	.070	2.6532	3.88715	.25727
.3	.233	1.4586	1.7490	.57177	.3	.070	2.6602	3.90704	.25596
.4	.227	1.4816	1.7731	.56400	.4	.069	2.6672	3.92695	.25467
.5	.222	1.5041	1.7971	.55647	.5	.069	2.6741	3.94681	.25338
.6	.217	1.5261	1.8210	.54915	.6	.068	2.6810	3.96652	.25212
.7	.213	1.5476	1.8449	.54204	.7	.068	2.6878	3.98629	.25087
.8	.208	1.5686	1.8687	.53518	.8	.068	2.6946	4.00594	.24964
.9	.204	1.5892	1.8925	.52841	.9	.067	2.7018	4.02561	.24841

5.	.200	1.6094	1.9162	.52188	15.	.067	2.7081	4.04529	.24721
.1	.196	1.6292	1.9398	.51553	.1	.066	2.7147	4.06493	.24601
.2	.192	1.6487	1.9633	.50937	.2	.065	2.7212	4.08463	.24482
.3	.188	1.6677	1.9867	.50336	.3	.965	2.7277	4.10433	.24364
.4	.185	1.6864	2.0101	.49748	.4	.065	2.7342	4.12401	.24248
.5	.182	1.7047	2.0335	.49178	.5	.064	2.7407	4.14360	.24134
.6	.179	1.7228	2.0567	.48619	.6	.054	1.7472	4.16310	.24021
.7	.175	1.7405	2.0799	.48079	.7	.664	2.7536	4.18260	.23908
.8	.172	1.7579	2.1030	.47550	.8	.063	2.7000	4.20213	.23797
.9	.169	1.7750	2.1261	.47034	.9	.063	2.7663	4.22163	.23687
6.	.167	1.79 8	2.14915	.46530	16.	.063	2.7726	4.24112	.23579
.1	.164	1.8083	1.17213	.46038	.1	.063	2.7788	4.26061	.23471
.2	.161	1.8245	2.19507	.45549	.2	.062	2.7850	4.28006	.23364
.3	.159	1.8405	2.21795	.45087	.3	.061	2.7911	4.29952	.23258
.4	.156	1.8563	2.24066	.44629	.4	.061	2.7972	4.31894	.23154
.5	.154	1.8718	2.26337	.44182	.5	.061	2.8034	4.33832	.23050
.6	.152	1.8871	2.28603	.43742	.6	.060	2.8094	4.35765	.22946
.7	.149	1.9021	2.30867	.43315	.7	.060	2.8154	4.37699	.22846
.8	.147	1.9169	2.33124	.42895	.4	.060	2.8213	4.39632	.22746
.9	.145	1.9315	2.35374	.42486	.9	.059	2.8273	4.41564	.22646
7.	.143	1.9159	2.37618	.42084	17.	.059	2.8332	4.43494	.225 9
.1	.141	1.9601	2.39857	.41692	.1	.058	2.8390	4.45424	.22452
.2	.139	1.9741	2.42090	.41307	.2	.058	2.8449	4.47349	.22355
.3	.137	1.9379	2.44319	.40930	.3	.058	2.8507	4.49274	.22258
.4	.135	2.0015	2.46543	.40561	.4	.057	2.8564	4.51298	.22163
.5	.133	2.0149	2.48764	.40199	.5	.057	2.8621	4.53121	.22068
.6	.131	2.0281	2.50982	.39841	.6	.057	2.8679	4.55041	.21974
.7	.130	2.0412	2.53189	.39495	.7	.056	2.8735	4.56956	.21881
.	.128	2.0541	2.55394	.39155	.8	.056	2.8792	4.58863	.21789
.9	.127	2.0669	2.57589	.38822	.9	.056	2.8848	4.60770	.21701
8.	.125	2.0794	2.59782	.38498	18.	.056	2.8904	4.62677	.21613
.1	.123	2.0919	2.61975	.38172	.1	.055	2.8959	4.64583	.21525
.2	.122	2.1041	2.64167	.37855	.2	.055	2.9014	4.66489	.21436
.3	.120	2.1163	2.66351	.37546	.3	.055	2.9069	4.68395	.21349
.4	.119	2.1282	2.68525	.37240	.4	.054	2.9123	4.70301	.21262
.5	.118	2.1401	2.70693	.36942	.5	.054	2.9178	4.72207	.21176
.6	.116	2.1518	2.72860	.36645	.6	.054	2.9231	4.74109	.21091
.7	.115	2.1633	2.75021	.36360	.7	.053	2.9285	4.76009	.21007
.8	.114	2.1748	2.77182	.36077	.8	.053	2.9338	4.77909	.20924
.9	.113	2.1861	2.79338	.35798	.9	.053	2.9391	4.79805	.20847
9.	.110	2.1972	2.81490	.35524	19.	.053	2.9444	4.81697	.20765
.1	.110	2.2083	2.83639	.35256	.1	.052	2.9497	4.83580	.20683
.2	.109	2.2192	2.85785	.34991	.2	.052	2.9540	4.85477	.20301
.3	.108	2.2300	2.87926	.34731	.3	.052	2.9601	4.87363	.20519
.4	.106	2.2407	2.90060	.34475	.4	.052	2.9652	4.89245	.20439
.5	.105	2.2513	2.92190	.34225	.5	.051	2.9703	4.91125	.20360
.6	.104	2.2618	2.94316	.33977	.6	.051	2.9755	4.93005	.20283
.7	.103	2.2721	2.96441	.33734	.7	.051	2.9806	4.94895	.20206
.8	.102	2.2824	2.98563	.33494	.8	.051	2.9856	4.96775	.20129
.9	.101	2.2925	2.00683	.33258	.9	.050	2.9907	4.98654	.20053
10.	.100	2.3026	3.02974	.33026	20.	.050	2.9957	5.00533	.19977
.1	.099	2.3125	3.04906	.32798	.1	.050	3.0007	5.02412	.19901
.2	.098	2.3223	3.07016	.32572	.2	.050	3.0056	5.04294	.19829
.3	.097	2.3321	3.09114	.32350	.3	.049	3.0106	5.06169	.19757
.4	.096	2.3418	3.11209	.32132	.4	.049	3.0154	5.08040	.19685
.5	.095	2.3514	3.13302	.31918	.5	.049	3.0210	5.09908	.19613
.6	.094	2.3308	3.15401	.31706	.6	.049	3.0252	5.11776	.19542
.7	.093	2.3702	3.17488	.31497	.7	.048	3.0301	5.13639	.19471
.8	.093	2.3795	3.19573	.31292	.8	.048	3.0349	5.15500	.19400
.9	.092	2.3887	3.21657	.31089	.9	.048	3.0397	5.17365	.19329

Example : — What must be the ratio of expansion with 90 pounds initial pressure to give a mean pressure of 45 ?

Ninety pounds gage is 105 pounds absolute.

$$105 \div 45 = 2.333.$$

In the column of initial per pound of mean effective the nearest value is 2.33124 and the corresponding ratio of expansion is 6.8.

TO FIND THE INITIAL PRESSURE.

Opposite the given point of cut-off or ratio of expansion, find the initial pressure required to produce a pound of mean pressure. Multiply this value by the given mean pressure and the product will be the absolute initial pressure required.

Example :—What initial pressure is required to give a mean pressure of 52.5 pounds at one-fifth cut-off ?

The initial per pound of mean pressure at one-fifth or .2 cut-off is shown by the table to be 1.9162 pounds.

$$1.9162 \times 52.5 = 100.6 \text{ lbs.}$$

Notice that we have computed here not the mean effective, but simply the mean pressure, the average forward pressure. This would be the mean effective only when there were absolutely no back pressure and the diagram was perfect like *a e f c g d*, Fig. 107 *d c* being the absolute zero of pressure. To get the mean effective even of a theoretical diagram you must subtract the absolute back pressure 14.7 or 15 pounds if the engine exhausts at atmospheric pressure, and the absolute pressure in the condenser if the engine runs condensing.

Remember it is the mean, not the mean effective pressure which you must use in working backward to get the other factors. To get a mean effective of 40 pounds, you must have a mean of 47, with a condensing, or 60 with a non-condensing engine making diagrams like Fig. 108.

THE EFFECT OF CLEARANCE.

The real ratio of expansion is always less than that indicated by the point of cut-off on account of the clearance. In Fig. 109 we have a diagram indicated by the solid lines in which the cut-off takes place at one fifth of the stroke, giving an apparent ratio of expansion of 5. Suppose the engine had 6⅔ per cent clearance and the diagram was 5 inches long, we must increase its length 6⅔ per cent to include the clearance volume. This would be

$.06\frac{2}{3} \times 5 = \frac{1}{3}$ of an inch.

Erecting the dotted clearance line one third of an inch from the admission line, we have a volume represented by the rectangle *a b c d* $1\frac{1}{3}$ inches wide which is expanded into a volume *a e f d* of $5\frac{1}{3}$ inches, giving a real ratio of expansion of

$$5\tfrac{1}{3} \div 1\tfrac{1}{3} = 4$$

The real ratio of expansion is the total volume of the cylinder including the clearance divided by the volume up to cut-off, also including clearance. This is the distance *d f* Fig. 109 divided by the distance *a b*. If we call the length of the working diagram *i f* unity so that the length of space added for clearance will be directly expressed by the clearance in per cent, the distance *d f* will be $1 + c$ and the distance a b will be $\frac{1}{r} + c$ when

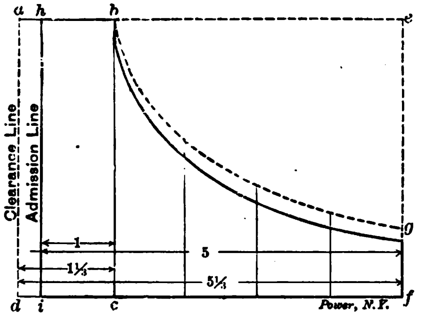


Fig. 109.

c = the clearance in fractions of the stroke

r = the apparent ratio of expansion.

Let R = the real ratio of expansion then

$$R = \frac{1 + c}{\frac{1}{r} + c}$$

TO FIND THE REAL RATIO OF EXPANSION.

GIVEN : — The percentage of clearance and the apparent ratio of expansion *r* or the fraction $\frac{1}{r}$ of the stroke completed at cut-of

RULE:—*Divide 1 plus the clearance by the sum of the clearance and the fraction of the stroke completed at cut-off, the clearance being expressed in fractions of the stroke.*

EXAMPLE:—What is the real ratio of expansion when steam is cut-off at one-fifth stroke and the clearance is six per cent?

Six per cent expressed as a fraction of the stroke is .06. The apparent ratio of expansion, r, = 5 then

$$\frac{1 + c}{\frac{1}{r} + c} = \frac{1 + .06}{\frac{1}{5} + .06} = \frac{1.06}{.26} = 4.08 \text{ nearly}$$

The actual ratios of expansion for various points of cut-off and percentage of clearance will be found in Table V.

TO FIND THE POINT OF CUT-OFF FOR A GIVEN REAL RATIO OF EXPANSION AND PERCENTAGE OF CLEARANCE.

By transposing the above formula we have

$$\frac{1 + c}{R} - c = \frac{1}{r}$$

RULE:—*Divide 1 plus the clearance by the real ratio of expansion. From the quotient subtract the clearance. The remainder will be the fraction of the stroke completed at cut-off.*

EXAMPLE:—At what fraction of the stroke must an engine cut-off in order to have a terminal pressure of 20 pounds absolute when the initial pressure is 110 pounds absolute the clearance being 5 per cent?

In order to have a terminal of 20 pounds 110 pound steam must be expanded to

$110 \div 20 = 5.5$ times its original volume.

Clearance $c = .05$. Real ratio $R = 5.5$

$$\frac{1 + C}{R} - c = \frac{1.05}{5.5} - .05 = .1409$$

The rules which have been given for mean effective pressure so far do not take the clearance into account. Suppose in the solid line of Fig. 110 we have a diagram from an engine cutting off at one-fifth stroke and with 6⅔ per cent. of clearance. Then as we have seen before the apparent ratio of expansion (*i. f.* divided by *h. b.*) will be 5, but the real ratio of expansion *d f* divided by *a b* = 4. If we compute the mean effective pressure for a ratio of 5 we shall be taking into account only the area *h b y f i*, whereas on account of the expansion of the steam in the clearance the expansion line ran higher along *b g* giving us the additional area *b g y*.

If, on the other hand, we compute the mean effective on a ratio of 4 we shall be taking into account the area *a b g f d*, which includes the clearance area *a h i d* and which refers to a stroke of *d. f.* while the stroke of the working diagram is *i f.* What we want to get is the mean effective pressure due to the area *h b g f i* which will be proportional to that area divided by its length.

To simplify the matter let us call the initial pressure unity.

TABLE V.

ACTUAL RATIOS OF EXPANSION AND MEAN PRESSURES PER POUND OF INITIAL PRESSURE FOR VARYING POINTS OF CUT-OFF AND PERCENTAGES OF CLEARANCE.

$$P_M = \frac{(s+c)\,[1 + \text{hyp. log. } R_A] - c}{s}$$

ACTUAL RATIO OF EXPANSION.

MEAN PRESSURE PER POUND OF ABSOLUTE INITIAL PRESSURE.*

Cut-off in Parts of Stroke. Frac.	Dec.	Clearance 0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%
1/10	.10	10.00	9.18	8.50	7.92	7.43	7.00	6.63	6.29	6.00	5.74	5.50	5.29	5.09	4.91	4.75	4.60
		.333	**.344**	**.357**	**.369**	**.381**	**.392**	**.402**	**.413**	**.423**	**.432**	**.441**	**.450**	**.458**	**.466**	**.474**	**.482**
2/19	.105	9.50	8.78	8.16	7.63	7.17	6.77	6.43	6.12	5.84	5.59	5.37	5.16	4.98	4.81	4.65	4.51
		.342	**.355**	**.367**	**.379**	**.390**	**.401**	**.412**	**.422**	**.431**	**.441**	**.450**	**.459**	**.467**	**.475**	**.482**	**.489**
1/9	.111	9.00	8.35	7.79	7.30	6.89	6.52	6.20	5.91	5.65	5.42	5.21	5.02	4.85	4.69	4.54	4.41
		.355	**.368**	**.380**	**.391**	**.402**	**.413**	**.423**	**.433**	**.442**	**.451**	**.450**	**.467**	**.476**	**.482**	**.491**	**.498**
2/17	.118	8.50	7.89	7.39	6.96	6.58	6.25	5.95	5.69	5.45	5.24	5.05	4.87	4.71	4.56	4.42	4.29
		.369	**.382**	**.394**	**.405**	**.416**	**.428**	**.436**	**.445**	**.457**	**.462**	**.471**	**.479**	**.467**	**.494**	**.501**	**.508**
1/8	.125	8.00	7.48	7.03	6.65	6.30	6.00	5.73	5.49	5.27	5.07	4.89	4.72	4.57	4.43	4.30	4.18
		.385	**.397**	**.403**	**.418**	**.429**	**.439**	**.448**	**.467**	**.467**	**.474**	**.482**	**.490**	**.497**	**.504**	**.511**	**.518**
2/15	.133	7.50	7.06	6.67	6.32	6.01	5.74	5.49	5.27	5.07	4.89	4.72	4.57	4.43	4.30	4.18	4.06
		.402	**.412**	**.424**	**.434**	**.444**	**.453**	**.462**	**.470**	**.479**	**.467**	**.494**	**.502**	**.510**	**.517**	**.524**	**.529**
1/7	.143	7.00	6.60	6.26	5.95	5.68	5.44	5.22	5.02	4.84	4.68	4.53	4.39	4.26	4.14	4.03	3.92
		.421	**.432**	**.442**	**.452**	**.460**	**.470**	**.479**	**.488**	**.495**	**.503**	**.510**	**.517**	**.524**	**.532**	**.537**	**.543**
2/13	.154	6.50	6.16	5.86	5.60	5.36	5.15	4.95	4.78	4.62	4.47	4.33	4.20	4.09	3.98	3.88	3.78
		.442	**.452**	**.462**	**.471**	**.480**	**.488**	**.496**	**.504**	**.512**	**.519**	**.526**	**.533**	**.540**	**.546**	**.553**	**.559**
1/6	.167	6.00	5.71	5.45	5.23	5.02	4.85	4.67	4.51	4.37	4.24	4.12	4.01	3.90	3.80	3.71	3.63
		.465	**.475**	**.484**	**.493**	**.501**	**.509**	**.517**	**.524**	**.531**	**.530**	**.545**	**.552**	**.553**	**.504**	**.509**	**.578**
2/11	.182	5.50	5.26	5.05	4.86	4.69	4.53	4.38	4.25	4.12	4.01	3.90	3.80	3.71	3.62	3.54	3.46
		.492	**.501**	**.509**	**.517**	**.525**	**.535**	**.542**	**.549**	**.555**	**.560**	**.566**	**.572**	**.578**	**.583**	**.589**	**.594**
3/16	.188	5.33	5.10	4.90	4.73	4.56	4.41	4.27	4.15	4.03	3.92	3.82	3.73	3.64	3.55	3.48	3.40
		.501	**.511**	**.518**	**.527**	**.534**	**.541**	**.547**	**.552**	**.562**	**.568**	**.574**	**.560**	**.535**	**.591**	**.597**	**.602**
1/5	.20	5.00	4.81	4.64	4.48	4.33	4.20	4.08	3.96	3.86	3.76	3.67	3.58	3.50	3.42	3.35	3.29
		.523	**.530**	**.538**	**.545**	**.552**	**.559**	**.566**	**.572**	**.578**	**.584**	**.590**	**.595**	**.601**	**.606**	**.611**	**.617**
2/9	.222	4.50	4.35	4.22	4.09	3.97	3.86	3.76	3.66	3.58	3.49	3.42	3.34	3.27	3.21	3.15	3.09
		.556	**.563**	**.570**	**.577**	**.583**	**.589**	**.595**	**.601**	**.607**	**.612**	**.618**	**.622**	**.628**	**.633**	**.637**	**.642**
1/4	.25	4.00	3.88	3.78	3.68	3.59	3.50	3.42	3.34	3.27	3.21	3.14	3.08	3.03	2.97	2.92	2.88
		.596	**.603**	**.609**	**.615**	**.621**	**.626**	**.631**	**.637**	**.641**	**.646**	**.650**	**.655**	**.660**	**.664**	**.668**	**.672**
3/10	.30	3.33	3.26	3.19	3.12	3.06	3.00	2.94	2.89	2.84	2.80	2.75	2.71	2.67	2.63	2.59	2.55
		.661	**.666**	**.671**	**.675**	**.679**	**.685**	**.688**	**.692**	**.697**	**.701**	**.705**	**.708**	**.712**	**.716**	**.719**	**.722**
5/16	.313	3.20	3.13	3.06	3.00	2.95	2.89	2.84	2.79	2.75	2.71	2.66	2.62	2.50	2.55	2.52	2.48
		.676	**.682**	**.685**	**.690**	**.695**	**.698**	**.702**	**.706**	**.710**	**.715**	**.717**	**.721**	**.725**	**.728**	**.732**	**.734**
1/3	.333	3.00	2.95	2.89	2.84	2.79	2.74	2.70	2.66	2.62	2.58	2.54	2.51	2.47	2.44	2.41	2.38
		.699	**.704**	**.703**	**.712**	**.716**	**.719**	**.722**	**.726**	**.731**	**.734**	**.737**	**.741**	**.743**	**.746**	**.749**	**.752**
3/8	.375	2.67	2.62	2.58	2.54	2.51	2.47	2.44	2.40	2.37	2.34	2.32	2.29	2.26	2.24	2.21	2.19
		.743	**.746**	**.749**	**.752**	**.757**	**.759**	**.763**	**.766**	**.768**	**.770**	**.775**	**.777**	**.778**	**.780**	**.783**	**.706**
2/5	.40	2.50	2.46	2.43	2.40	2.36	2.33	2.30	2.28	2.25	2.22	2.20	2.18	2.15	2.13	2.11	2.09
		.767	**.769**	**.772**	**.776**	**.778**	**.781**	**.784**	**.787**	**.789**	**.791**	**.794**	**.797**	**.799**	**.801**	**.803**	**.805**
7/16	.438	2.29	2.26	2.23	2.20	2.18	2.15	2.13	2.11	2.09	2.06	2.04	2.03	2.01	1.99	1.97	1.96
		.799	**.803**	**.805**	**.807**	**.810**	**.812**	**.814**	**.817**	**.819**	**.820**	**.823**	**.820**	**.827**	**.828**	**.830**	**.833**
1/2	.50	2.00	1.98	1.96	1.94	1.93	1.91	1.89	1.88	1.86	1.85	1.83	1.82	1.81	1.79	1.78	1.77
		.846	**.848**	**.850**	**.852**	**.854**	**.856**	**.837**	**.860**	**.861**	**.863**	**.863**	**.865**	**.867**	**.868**	**.869**	**.871**
9/16	.563	1.78	1.76	1.75	1.74	1.73	1.71	1.70	1.69	1.68	1.67	1.66	1.65	1.64	1.63	1.62	1.61
		.885	**.886**	**.883**	**.891**	**.891**	**.891**	**.893**	**.894**	**.896**	**.897**	**.898**	**.899**	**.900**	**.901**	**.902**	**.902**
3/5	.60	1.67	1.66	1.65	1.64	1.63	1.62	1.61	1.60	1.59	1.58	1.57	1.56	1.56	1.55	1.54	1.53
		.906	**.909**	**.909**	**.911**	**.913**	**.914**	**.914**	**.915**	**.915**	**.916**	**.916**	**.917**	**.919**	**.920**	**.920**	**.920**
5/8	.625	1.60	1.59	1.58	1.57	1.56	1.56	1.55	1.54	1.53	1.52	1.52	1.51	1.50	1.50	1.49	1.48
		.919	**.919**	**.920**	**.921**	**.923**	**.925**	**.925**	**.925**	**.926**	**.927**	**.928**	**.028**	**.929**	**.930**	**.930**	**.930**
2/3	.667	1.50	1.49	1.49	1.48	1.47	1.46	1.46	1.45	1.45	1.44	1.43	1.43	1.42	1.42	1.41	1.41
		.937	**.937**	**.939**	**.940**	**.940**	**.940**	**.942**	**.942**	**.944**	**.944**	**.944**	**.945**	**.945**	**.946**	**.940**	**.948**
11/16	.688	1.45	1.45	1.44	1.44	1.43	1.42	1.42	1.41	1.41	1.40	1.40	1.39	1.39	1.38	1.38	1.37
		.944	**.947**	**.947**	**.948**	**.948**	**.948**	**.949**	**.949**	**.952**	**.952**	**.953**	**.931**	**.954**	**.951**	**.952**	**.952**
7/10	.70	1.43	1.42	1.42	1.41	1.41	1.40	1.40	1.39	1.39	1.38	1.38	1.37	1.37	1.36	1.36	1.35
		.949	**.950**	**.949**	**.951**	**.952**	**.952**	**.953**	**.953**	**.954**	**.954**	**.955**	**.955**	**.956**	**.956**	**.956**	**.957**
3/4	.75	1.33	1.33	1.33	1.32	1.32	1.31	1.31	1.31	1.30	1.30	1.29	1.29	1.29	1.28	1.28	1.28
		.964	**.967**	**.967**	**.957**	**.969**	**.969**	**.969**	**.969**	**.969**	**.970**	**.970**	**.970**	**.970**	**.970**	**.970**	**.970**

* To obtain actual MEAN EFFECTIVE PRESSURE, multiply the Absolute Initial Pressure by the tabular quantity in **Bold-Faced Type** corresponding to the Cut-off and Clearance, and subtract from the product 7 pounds for Condensing Engines and 20 pounds for Non-Condensing Engines.

We will find thus the mean effective per pound of initial pressure and by multiplying this by the initial can find the mean effective for any initial pressure.

We can easily find the area of the whole diagram for if we call

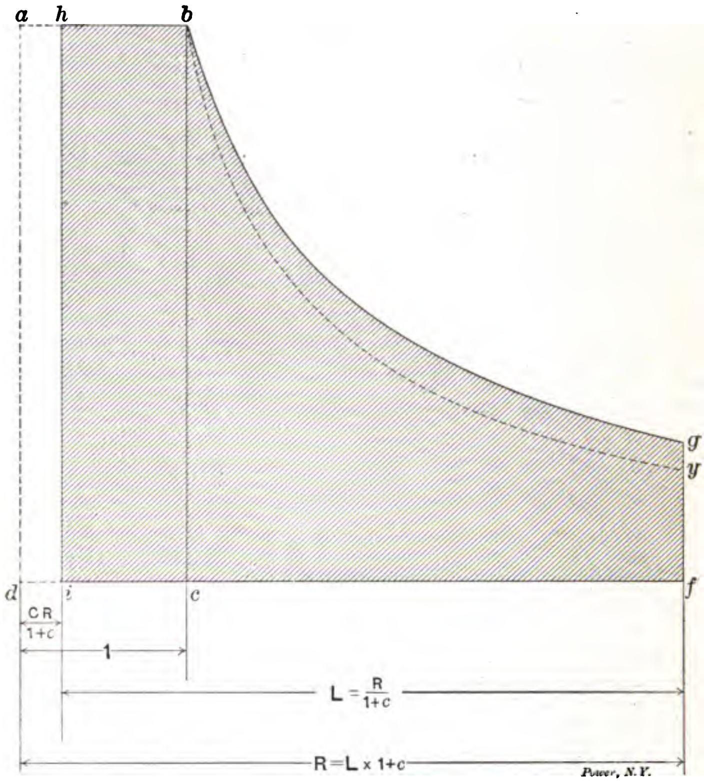


Fig. 110.

a b c d 1 *b g f c* will be the hyperbolic logarithm of the ratio of expansion so that the area *a b g f d* will be

$$1 + \text{hyp. log. R}$$

when R. is the actual ratio of expansion.

In this case R = 4, *a b c d* = 1 and *b g f c* = 1.3863 just as in Fig. 107.

The total area

$$1 + \text{hyp. log. R} = 1 + 1.3863 = 2.3863.$$

If we call *a b* or *c d* unity the length of this total diagram is

equal to the ratio of expansion for if $d\,c = 1$, $d\,f$ must equal as many times 1 as the final volume $d\,f$ is times the initial volume $d\,c$. Calling the length $i\,f$ of the working diagram L then the length $d\,f$ of the total diagram is also

$$L \times 1 + c$$

therefore

$$R = L \times 1 + c$$

$$L = \frac{R}{1 + c}$$

The length $d\,i$ of the clearance space, is the stroke L multiplied by the percentage of clearance. Using the above value for the stroke we have for the length $d\,i$ of the clearance space

$$\frac{c\ R}{1 + c}$$

since its height is unity this value represents its area and this is to be subtracted from the total area giving us

$$1 + \text{hyp. log. } R - \frac{c\ R}{1 + c}$$

for the area $h\,b\,g\,f\,i$.

The mean pressure will be this area divided by its length which length L as we have seen equals

$$\frac{R}{1 + c}$$

We have then for the mean pressure per pound of initial of the diagram corrected for clearance, that is of the ideal diagram $h\,b\,g\,f\,i$ in Fig. 110.

$$\frac{1 + \text{hyp. log. } R - \dfrac{c\ R}{1 + c}}{\dfrac{R}{1 + c}} =$$

$$\frac{1 + \text{hyp. log. } R - \dfrac{c\ R}{1 + c} \times 1 + c}{R} =$$

$$\left(\frac{1 + \text{hyp. log. } R}{R} \times 1 + c\right) - c$$

This is the mean per pound of initial pressure for any given absolute initial pressure P_a the mean effective pressure of the ideal diagram corrected for clearance and without counter pressure would be

$$M E P = P_a \left(\frac{1 + \text{hyp. log. } R}{R} \times 1 + c - c \right)$$

where R = the real ratio of expansion or, substituting $\frac{1 + c}{\frac{1}{r} + c}$ for R where r = the apparent ratio of expansion and $\frac{1}{r}$ the fraction of the stroke completed at cut-off

$$M E P = P_a \left(\frac{1 + \text{hyp. log. } \frac{1 + c}{\frac{1}{r} + c}}{\frac{1 + c}{\frac{1}{r} + c}} \times 1 + c - c = \right.$$

$$P_a \left(1 + \text{hyp. log. } \frac{1 + c}{\frac{1}{r} + c} \times \frac{1}{r} + c - c \right)$$

TO FIND THE MEAN PRESSURE OF THE IDEAL DIAGRAM CORRECTED FOR CLEARANCE.

GIVEN :—Fraction of stroke completed at cut-off, and percentage of clearance.

RULE :—*Divide 1 plus the clearance by the clearance plus the fraction of the stroke completed at cut-off.*

Multiply 1 plus the hyperbolic logarithm of the quotient by the fraction of the stroke completed at cut-off plus the clearance, and subtract the clearance from the product. The remainder will be the mean per pound of initial pressure.

Multiply this remainder by the absolute initial pressure and the product will be the mean pressure of the ideal diagram corrected for clearance.

EXAMPLE :—What is the mean effective pressure of the ideal diagram made by an absolute initial of 100 pounds cut-off at one-fifth stroke in a cylinder having 4 per cent. clearance.

clearance = c .04

fraction of stroke completed at cut-off $\frac{1}{r}$ = .2

absolute initial pressure P_a = 100

$$\frac{1 + c}{\frac{1}{r} + c} = \frac{1 + .04}{.2 + .04} = \frac{1.04}{.24} = 4.33$$

hyp. log. 4.33 (from table) = 1.4656

$$\left(1 + \text{hyp. log. } 4.33 \times \frac{1}{r} + c\right) - c = 2.4656 \times .24 - .04$$

$$\left(1 + \text{hyp. log. } 4.33 \times \frac{1}{r} + c\right) - c =$$

$$\left(1 + 1.4656 \times \frac{1}{5} + .04\right) - .04 =$$

$$2.4656 \times .24 - .04 = 551744$$

This is the mean per pound of initial. For 100 pounds initial the mean effective would be

$$100 \times .551744 = 55.1744$$

The values per pound of initial pressure for various points of cut-off and percentage of clearance will be found in Table V.

CHAPTER XV.

STEAM CONSUMPTION FROM THE DIAGRAM.

Knowing the cubic capacity of the cylinder and the number of times it is filled and emptied per hour, we can compute the cubic feet of steam passing through the engine in that time. Knowing from the diagram the pressure of this steam we can find in a steam table the weight per cubic foot, and thus the weight of steam passed per hour. Table VI gives the weights of steam per cubic foot for the different pressures. The diagram also gives us a measure of the horse-power, dividing by which we get the number of pounds of steam accounted for by the diagram per hourly horse-power. This will be always less than the actual amount of steam supplied to the engine, because a considerable proportion of such steam is condensed on its entrance to the cylinder, and is not re-evaporated until after the valve opens for exhaust, so that it does not show as steam upon the diagram at all. The computation is of use, however, for purposes of comparison, and as a measure of the minimum amount of steam which the diagram would allow per horse-power, and should be understood by one who desires to attain proficiency with the indicator.

Let $A =$ area of piston in square inches,

$S =$ length of stroke in feet,

$N =$ number of strokes per minute,

$P =$ mean effective pressure, indicated by diagram.

The volume generated by the piston per hour would be

$$\frac{A}{144} \times S \times 60\, N$$

the area in square inches divided by 144 to reduce to square feet, multiplied by the length of the stroke, gives the cubic feet per

stroke, and by 60 times the number of strokes per minute gives the number of cubic feet passed through by the piston per hour.

The horse-power is $\frac{PANS}{33,000}$

Dividing by this value we get the number of cubic feet passed through by the piston in an hour for each horse-power. As the area, length of stroke, and number of strokes per minute are used in calculating both the volume and the horse-power they cancel each other in the division, and the formula becomes

$$\frac{\frac{A}{144} S\,60\,N}{\frac{PANS}{33,000}} = \frac{A\,S\,60\,N\,33,00}{144\,PANS} = \frac{13,750}{P}$$

or in plain language, 13,750 divided by the mean effective pressure will give the cubic feet of piston displacement per hour for each horsepower generated by any engine, whatever its size or speed. Substituting for P the common abbreviation of the mean effective pressure we have

$$\frac{13,700}{\text{M. E. P.}} = \text{volume generated per hour per horse-power} \qquad (1)$$

If the engine had no clearance nor compression and the release did not occur until the end of the stroke we could measure the pressure of the steam at the point a, Fig. 111, find in a steam table the weight of steam of that pressure per cubic foot, multiply the volume per horse-power by that weight, and find the weight per horse-power per hour. As the quantities in the steam tables are usually given in pounds absolute it is better to measure from the zero line $o\,x$, or to add 14.7 pounds to the measurement from the atmospheric line. Or we could equally well measure the pressure at any other point after the cut-off valve closes, and take such proportion of the volume given by formula 1, based on the whole stroke, as the portion of the

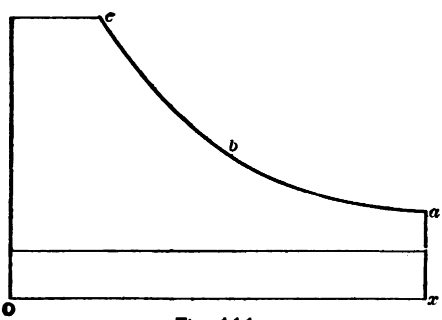


Fig. 111. *Power, N.Y.*

stroke completed by the piston up to the point chosen bears to the full stroke. If we measure the volume at *a* we have had so many complete cylinderfuls of steam at that pressure, and formula 1 will give us the volume per horse-power. If we measure the volume at half-stroke *b* we have had only one-half the volume at this higher pressure, and formula 1 must be multiplied by .5 to give the number of cubic feet per horse-power per hour of this higher pressure steam. Likewise if we measure the pressure at one-quarter stroke *c* we shall have had but one-quarter of the volume, and must multiply the formula by .25, and so for any other fraction of the stroke. If the amount of steam in the cylinder were constant throughout the expansion the weight per horse-power per hour would be the same whether we measured it at cut-off or at release, or at any point between, but condensation and re-evaporation are going on, so that there is more steam in the cylinder later in the stroke than immediately after the cut-off, and there will ususually be found to be a greater amount of steam accounted for per horse-power per hour the nearer the measurements are made to the point of release. Call the fraction of the stroke completed at the point chosen F, and the weight of steam per cubic foot at that pressure w_f, then under the simple conditions assumed

$$\frac{13{,}750}{\text{M.E.P.}} F w_f = \text{lbs. steam per h. p. per hour} \qquad [2]$$

when the pressure is measured at the end of the stroke, as at *a*, F becomes unity or one, and the formula becomes

$$\frac{13{,}750 w}{\text{M.E.P.}} = \text{lbs. steam per h. p. h.} \qquad [3]$$

We have yet to determine the amount of steam required to fill the clearance, and the amount left in the cylinder when the exhaust valve closes. As we cannot exhaust into a perfect vacuum there will always be some of the latter steam, even when there is no compression. Suppose the engine to have 5 per cent clearance, then when the piston was at *a* instead of having the volume swept through by the piston behind it we should have 1.05 times that volume. When the piston was at half-stroke we should have instead of .5 of the piston displacement .55 of that volume, and generally for any fraction F of the stroke completed at the point chosen for measurement we should have $F + c$ of

the piston displacement behind it, c being the clearance in fractions of the stroke, and the steam per horse-power per hour becomes

$$\frac{13{,}750}{\text{M. E. P.}}\,(F + c)\,w_f$$

Suppose in Fig. 112 the exhaust valve to close at e when the return stroke was .8 completed. The volume of steam shut in would be the area of the piston in square feet multiplied by the

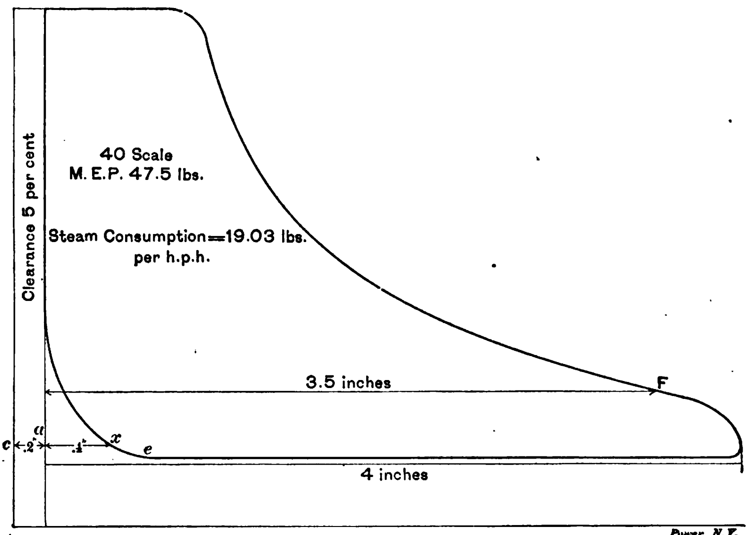


Fig. 112.

fraction of stroke uncompleted plus the five one-hundredths of the stroke included in the clearance, that is, .2 + .05 = .25; or generally, calling the portion of the stroke uncompleted at the compression x, the volume inclosed would be per hour

$$\frac{A}{144}\,s\,(x + c)\,60\,N \qquad [4]$$

and this divided by the horse-power $\frac{PANS}{33{,}000}$ to give the volume saved per horse-power, and multiplied by the weight w_x of steam per cubic foot at the pressure obtained at the point x would be

$$\frac{13{,}750}{\text{M. E. P.}}\,(x + c)\,w_x$$

Subtracting this from formula 4 we have

$\frac{13,750}{\text{M. E. P.}}$ $[\,(F + c)\, w_f \,-\, (x + c)\, w_x\,]$ = steam accounted for per h. p. per hour.

Where c = clearance in fractions of the stroke,

F = fraction of stroke completed at point chosen on expansion line,

X = fraction of stroke uncompleted at point chosen on compression line,

w_f = wt. per cu. ft. of steam at pressure measured at F,

w_x = wt. per cu. ft. of steam at pressure measured at x.

RULE:—*To the fraction of the forward stroke completed at the point chosen add the clearance, also in fractions of the stroke, and multiply the sum by the weight per cubic foot of steam of the pressure measured at this point.* (*Product 1.*)

To the fraction of the return stroke uncompleted at the point chosen on the compression line add the clearance, expressed as before, and multiply the sum by the weight per cubic foot of steam of the pressure measured at this point. (*Product* 2.)

Multiply the difference between products 1 and 2 by the quotient of 13,750 divided by the M. E. P.; the final product will be the number of pounds of steam per hour per horse-power accounted for by the indicator.

As an assistance in working with the above rule or formula Table VI gives the value of 13,750 divided by mean effective pressures of from 10 to 100 pounds. The first column under zero gives the quotients for even pounds, the succeeding columns for additional tenths of pounds. Thus the quotient of $\frac{13,750}{35.6}$ would be found in the horizontal line with 35 and in the column under 6 to be 386.23.

EXAMPLE: The diagram shown in Fig. 112 shows with a 40 scale a M. E. P. of 47.5 pounds; clearance 5 per cent. How much steam is accounted for per horse power per hour?

Let us select the points F and x from which to make our measurements. The whole length of the diagram is 4 inches, the length to the point F, 3.5 inches. The fraction F of the stroke completed at this point is therefore $\frac{3.5}{4} = .875$. The distance $x\,a$ equals .4 of an inch, and the fraction of the return stroke

uncompleted at the point x is $\frac{.4}{4} = .1$

The pressure (absolute) at F is 32 pounds, at x 19 pounds. The weight of steam per cubic foot at 32 pounds is .0789, at 19 pounds .0483

$$\begin{aligned} \text{then } c &= .05 \\ F &= .875 \\ x &= .1 \\ w_f &= .0789 \\ w_x &= .0483 \\ \text{and M. E. P.} &= 47.5 \end{aligned}$$

The steam accounted for per horse-power per hour is

$$\frac{13,750}{47.5} \times [(.875 + .05)\ .0789 - (.1 + .05)\ .0483].$$

From the table we find the value of $\frac{13,750}{47.5}$ to be 289.47 and we have $289.47 \times [(.925 \times .0789) - (.15 \times .0483)] = 19.03$ pounds of steam per hour for each horse-power.

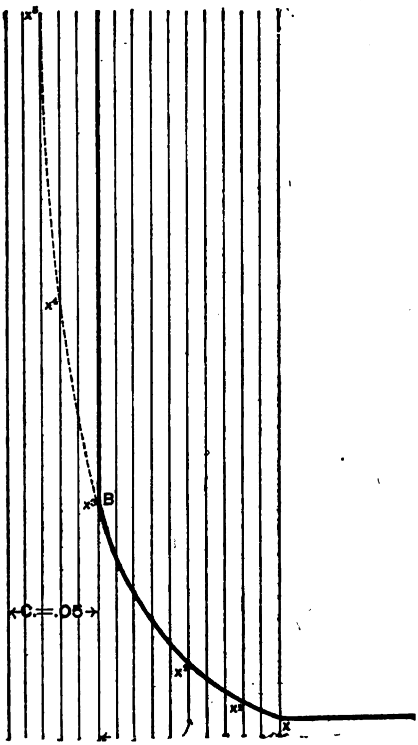


Fig. 113.

It is not necessary that the point X at which the pressure of thesteam saved by compression is measured shall be at the commencement of compression. It may be located at any point upon that line or upon the dotted continuation of that line into the clearance space. In Fig. 113, representing the compression corner of a diagram on a large scale, let the vertical divisions represent hundredths of the stroke, the clearance C being five per cent or five-hundredths, and the exhaust valve closing at X when ten one-hundredths of the stroke are uncompleted. When the

TABLE VI.

Values of $\frac{13750}{\text{M.E.P.}}$

TABLE FOR COMPUTING STEAM CONSUMPTION.

	0	1	2	3	4	5	6	7	8	9
10	1375.00	1361.39	1348.04	1334.95	1322.15	1309.52	1297.17	1285.04	1273.14	1261.46
11	1250.00	1238.74	1227.68	1216.81	1206.13	1195.65	1185.34	1175.19	1165.25	1155.46
12	1145.83	1136.36	1127.05	1117.88	1108.87	1100.00	1091.11	1082.67	1074.21	1062.01
13	1057.69	1049.62	1041 66	1033.83	1026.12	1018 51	1011.03	1003.64	996.38	989.21
14	982.14	975.18	968.31	961.54	954.86	948.29	941.78	935.37	929.00	922.82
15	916.67	910.60	904.61	898.69	893.05	867.09	881.41	875.79	870.25	864.77
16	871.87	854.04	848.76	843.55	838.41	833.33	828.31	823.35	818.45	813.61
17	808.82	804.09	799.42	794.79	790.23	785.71	781.25	776.84	772.47	768.15
18	763.89	759.67	755.49	751.36	747.28	743.24	739.24	735.29	731.38	727.51
19	723.68	719.89	716.15	712.43	708.76	705.13	701.53	697.99	694.44	690.95
20	687.50	683.08	680.69	677.34	674.02	670.73	667.47	664.25	661.06	657.84
21	654.76	651.66	648.58	645.54	642.52	639.53	636.57	633.64	630.73	627.85
22	625.00	622.17	619.37	616.59	613.94	611.11	608.41	605.72	603.07	600.43
23	597 83	595.24	592.67	590.12	587.61	585.11	582.62	580.16	577.73	575.31
24	572.92	570.54	568.18	565.84	563.52	561.22	558.94	556.67	554.43	552.21
25	550.00	547.81	545.64	543.47	541.33	539.21	537.11	535.02	532.94	530.88
26	528.85	526.82	524.81	522.81	520.83	518.87	516.91	514.98	513.06	511.15
27	509.26	507.38	505.51	503 66	501.82	500.00	498.11	496.39	494.60	493.19
28	491.07	489.32	487.55	485.86	484.15	482.45	480.76	479.09	477.43	476.12
29	474.14	472.51	470.89	469.28	467.68	466.10	464.53	462.89	461.40	459.86
30	458.33	456.81	455.30	453.79	452.30	450.82	449.34	447.88	446.42	444.98
31	443.55	442.12	441.99	439.30	437.83	436.51	435.12	433.75	432.39	431.35
32	429.69	428.35	427.01	425.69	424.38	423.07	421.77	420.49	419.21	417.93
33	416.67	415.41	413.85	412.91	411.67	410.44	409.22	408.01	406.80	405.60
34	404.41	403.22	402.05	400.87	399.71	398.55	397.39	396.25	395.11	393.98
35	392.84	391.73	390.63	389.51	388.41	387.32	386.23	385.15	384.08	383.01
36	381.94	380.89	379.83	378.78	377.75	376.71	375.68	374.66	373.64	372.62
37	371.62	370.62	369.62	368.63	367.65	366.66	365.69	364.72	363.75	362.79
38	361.84	360.89	359.94	359.00	358.07	357.40	356.22	355.29	354.38	353.47
39	352.56	351.64	350.77	349.87	348.98	348.10	347.22	346.34	345.47	344.11
40	343.75	342.89	342.32	341.19	340.34	339.51	338.67	337.83	337.01	336.18
41	335.36	334.55	333.74	332.92	332.12	331.32	330.52	329.71	328.94	328.16
42	327.38	326.36	325.83	325.06	324.26	323.50	322.77	322.01	321.35	320.51
43	319.77	319.02	318.29	317.55	316.82	316 09	315.36	314.64	313.92	313.21
44	312.50	311.79	311.09	310.38	309.68	308.98	308.29	300.61	306.92	306.23
45	305.55	304.88	304.20	303.55	302.86	302.19	301.53	300.87	300.22	299.34
46	298.91	298.26	297.62	296.97	296.33	295.48	295.06	294.43	293.80	292.96
47	292.55	291.93	291.31	290.61	290.08	289.47	288.86	288.26	287.65	287.05
48	286.46	285.86	285.26	284.66	284.09	283.50	282.92	282.34	281.76	281.18
49	280.61	280.04	279.47	278.09	278.34	277.77	277.21	276.66	276.10	275.55
50	275.00	274.45	273.90	273.35	272.82	272.27	271.73	271.20	270.67	270.13
51	269.61	269.08	268.55	268.03	267.51	266.99	266.47	265.95	265.44	264.93
52	264.43	263.91	263.41	262.91	262.40	261.90	261.40	260.91	260.41	258.03
53	259.43	258.94	258.45	257.97	257.49	257.00	256.53	256.05	255.57	255.10
54	254.63	254.16	253.69	253.22	252.75	252.29	251.83	251.37	250.91	250.47
55	250.00	249.54	249.09	248.64	248.19	247.74	247.30	246.86	246.41	245.97
56	244.64	245.10	244.66	244.22	243.79	243.36	242.93	242.50	242.07	241.65
57	241.23	240.80	240.38	239.26	237 80	239.13	238.71	238.30	237.88	237.47

TABLE FOR COMPUTING STEAM CONSUMPTION. (Continued)

	0	1	2	3	4	5	6	7	8	9
58	233.62	236.66	236.25	235.84	235.44	235.04	234.64	234.22	233.84	233.44
59	237.07	232.64	232.26	231.87	231.48	231.09	230.71	230.31	229.93	229.54
60	229.17	228.79	228.41	228.03	227.65	227.27	226.89	226.52	226.15	225.78
61	225.41	225.04	224.67	224.30	223.92	223.57	223.21	222.85	222.49	222.13
62	221.71	221.42	221.06	220.67	220.35	220.00	219.64	219.29	218.93	218.60
63	218.25	217.91	217.56	217.21	216.87	216.53	216.19	215.06	215.51	215.18
64	214.84	214.50	214.17	213.99	213.50	213.17	212.69	212.51	212.19	211.86
65	211.54	211.21	210.88	210.56	210.44	209.92	209.60	209.28	208.96	208.64
66	208.31	208.01	207.70	207.39	207.08	206.70	206.45	206.14	205.83	205.53
67	205.22	204.91	204.61	204.31	204.00	203.70	203.40	203.10	202.80	202.50
68	202.20	201.91	201.61	201.32	201.04	200.73	200.43	200.14	199.85	199.56
69	199.27	198.98	198.69	198.41	198.12	197.84	196.12	197.56	196.99	196.70
70	196.43	196.14	195.86	195.59	195.31	195.03	194.75	194.34	194.21	193.93
71	193.66	193.39	193.12	192.84	192.57	192.31	192.03	191.77	191.50	191.23
72	190.97	190.71	190.44	190.17	189.91	189.65	189.39	189.13	188.87	187.24
73	188.36	188.10	187.84	187.58	187.33	187.07	186.82	186.56	186.31	186.06
74	185.80	185.56	185.30	185.06	184.81	184.56	184.31	184.07	183.82	183.57
75	183.34	183.09	182.84	182.60	182.36	182.11	181.87	181.63	181.39	181.16
76	180.92	180.68	180.45	180.21	179.97	179.73	179.50	179.27	179.03	178.80
77	178.57	178.34	178.11	177.87	177.65	177.42	177.19	177.09	176.73	176.51
78	176.28	176.05	175.83	175.61	175.38	175.16	174.81	174.71	174.49	174.27
79	174.05	173.83	173.61	173.39	173.17	172.95	172.73	172.52	172.18	172.09
80	171.87	171.66	171.45	171.23	171.02	170.81	170.59	170.38	170.17	169.96
81	169.75	169.54	169.33	169.12	168.91	168.71	168.50	168.29	168.09	167.88
82	167.68	167.47	167.27	167.07	166.86	166.67	166.46	166.26	166.06	165.86
83	165.66	165.46	165.26	165.06	164.86	164.67	164.47	164.27	164.09	163.88
84	163.69	163.49	163.30	163.11	162.92	162.72	162.52	162.22	162.14	161.96
85	161.76	161.57	161.38	161.19	161.01	160.82	160.63	160.44	160.25	160.07
86	159.88	159.70	159.51	159.33	159.14	158.73	158.77	158.59	158.41	158.23
87	158.04	157.86	157.68	157.50	157.32	157.14	156.96	156.78	156.61	156.44
88	156.25	156.07	155.89	155.71	155.54	155.36	155.19	155.01	154.84	154.66
89	154.49	154.32	154.14	153.97	153.80	153.63	153.46	153.29	153.12	152.94
90	153.78	152.61	152.44	152.27	152.10	151.93	151.76	151.60	151.54	151.26
91	151.09	150.93	150.77	150.60	150.43	150.27	150.11	149.94	149.77	149.61
92	149.45	149.29	149.13	148.97	148.81	148.64	148.48	148.32	148.16	148.00
93	147.85	147.58	147.53	147.25	147.21	147.05	146.90	146.73	146.59	146.43
94	146.27	146.12	145.96	145.81	145.65	145.50	145.34	145.19	145.04	144.89
95	144.73	144.58	144.48	144.28	144.13	143.98	143.82	143.67	143.52	143.37
96	143.23	143.08	142.93	142.67	142.63	142.48	142.34	142.19	142.04	141.90
97	141.75	141.61	141.46	141.31	141.17	141.02	140.88	140.73	140.59	140.44
98	140.31	140.17	140.02	139.87	139.73	139.59	139.46	139.31	139.17	139.03
99	138.88	138.74	138.61	138.46	138.33	138.19	138.05	137.91	137.77	137.63
100	137.50	137.36	137.22	137.09	136.95	136.81	136.68	136.54	136.41	136.27

exhaust valve closes we have a volume of steam enclosed equal to $C + X = .05 + .10 = .15$ of the displacement at the pressure X, or if we measure at X^1, when .08 of the stroke remain to be completed, we shall have $.05 + .08 = .13$ at the pressure X^1, or .10 at the pressure X^2 or .05 at the pressure X^3 .03 at X^4, etc., so that so long as we measure the pressures and volumes accordingly X may be located anywhere on the expansion curve, or even on the dotted extension of that line inside the clearance space. The compression after the piston has reached the end of its stroke will go on by the admission of the higher pressure steam. Suppose in Fig. 114 the exhaust valve closes at E, shutting in a vol-

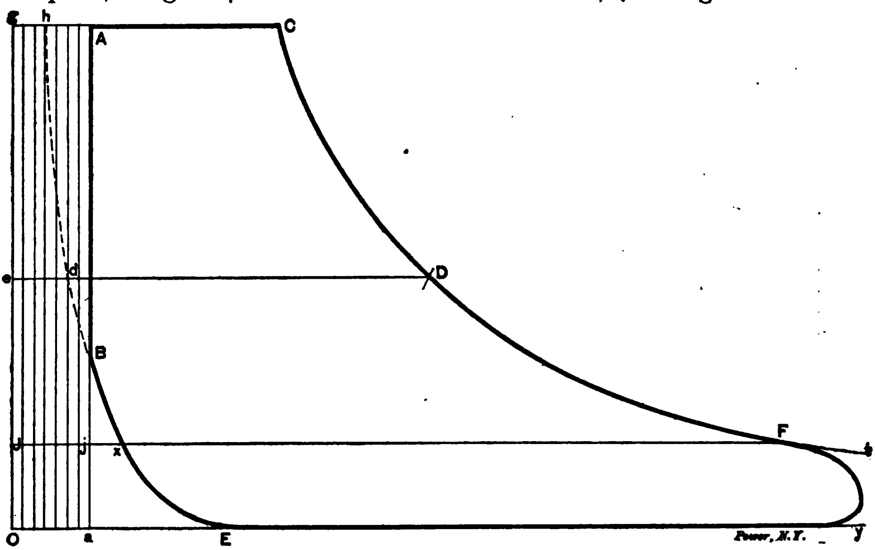


Fig. 114.

ume proportional to the line $O\ E$ of exhaust steam. When the piston reaches the end of its stroke on the line $A\ a$ the clearance will be full of steam raised by compression to the pressure B. The admission valve being now opened, live steam rushes in and raises the pressure to that of the steam line $A\ C$, by which process the steam saved by compression and which occupied the whole clearance at a pressure B before admission is compressed to a volume proportional to the line $g\ h$, corresponding with the pressure to which it is subjected. At this pressure, it will be seen, it occupies three-sevenths of the clearance space, and the remain-

ing four-sevenths must be supplied from the boiler. The amount of new steam supplied up to the point of cut-off then is proportional to the line *h C*. When the pencil reached *D* the compression steam had expanded to a volume proprotional to *e d*, corresponding with that pressure, and the new steam involved in the stroke is proportional to the line *D d*, and this is true of any line drawn horizontally across the diagram between the expansion and compression line, or the continuation of the latter into the clearance. This fact, when the compression is such that a horizontal line from the point which we wish to use on the expansion line will cut the compression line, as *F x*, gives a simple process for finding the steam accounted for by the indicator corrected both for clearance and compression. It will be remembered that the formula when the whole volume of the displacement was involved and the pressure taken at the end of the stroke *t* was by formula [1]

$$\frac{13750\ w_t}{\text{M. E. P.}}$$

where w_t was the weight per cubic foot of steam at the terminal pressure. If instead of measuring the pressure at the terminus of the stroke *t*, we take any other time point, as *F* or *D*, the volume involved will be to the whole displacement volume as *x F* or *d D* is to the length of the diagram *a y*. If as before $F =$ the fraction of the stroke completed at the point chosen for measurement, as *F*, Fig. 114 and $X =$ the portion of the return stroke uncompleted at the point chosen on the compression line, then $F—X$ (i.e., *j F—j X*, Fig. 114) will be the fraction of the whole length of the diagram occupied by the line *X F*, included between the expansion and compression lines. Substituting in formula [1] w_f = the weight per cubic foot at the pressure measured at point *F* for w_t, and muliplying by the fraction $F—X$, we get the steam accounted for per horse-power and per hour, reducing the complete formula to

$$\frac{13750}{\text{M.E.P.}}(F-X)\ wf$$

RULE:—*From the fraction of the stroke completed at the point chosen on the expansion line subtract the fraction of the stroke uncompleted at the point on the compression line which is in the same horizontal line. Multiply the difference by the weight per*

cubic foot of steam at the pressure measured at the points chosen and by the quotient of 13,750 divided by the mean effective pressure The final product will be the weight of steam accounted for per horse-power per hour.

When the terminal pressure is so high or the compression is so small that a horizontal line would cut the admission rather than the compression line, the point X will be independently located and the formula used on p. 136. rather than to construct the extension of the compression line into the clearance, though the simple method just described would still be used on speculative or theoretical work. If the horizontal line intersects the junction of the compression and admission lines as at B the portion X of the stroke uncompleted at this point becomes zero. If the horizontal line crosses the admission line, as at D d, X becomes minus, and the distance from the admission line A a to the point d where the horizontal crosses the compression line must be added to F. The value F--X, however, would in this case be easier arrived at and may be found in any case by dividing the length of the horizontal line, as d D, included between the expansion lines by the length of the diagram a y.

Draw a line across the diagram parallel with the atmospheric line. Divide the length of that portion of this line included between the expansion and compression lines by the extreme length of the diagram, and multiply the quotient by the weight per cubic foot of steam at the pressure indicated by the height of the horizontal line. Multiply this product by the quotient of 13,750 divided by the mean effective pressure, and the result will be the pounds of steam accounted for per horse-power per hour.

This rule is identical with the other, the proportion of the line of quantities to the length of the diagram being arrived at differently. It can be deduced from the formula algebraically as follows: When the points F and X are at the same height $w_x = w_f$ and the formula becomes

$$\frac{13,750}{\text{M. E. P.}} \left[(F + \text{C})\, w_f - (X + C)\, w_f \right] =$$

$$\frac{13,750}{\text{M. E. P.}} (F - \text{X})\, w_f$$

STEAM ACCOUNTED FOR BY MULTIPLE-CYLINDER DIAGRAMS.

We have seen that the amount of steam in the cylinder is different at different points in the stroke, increasing by re-evaporation as we get nearer the release. The same thing holds true in a multiple cylinder engine. A portion of steam is measured off by the cut-off valve of the high pressure cylinder. This portion in passing through the series of cylinders develops a determined amount of power. If the quantity of steam remained constant the quantity per horse-power hour would be the same whether we measured it immediately on the closure of the high pressure cut-off valve or just before its final release in the low pressure cylinder. But its quantity is constantly changing and we shall find more steam accounted for per horse-power hour at the terminal end of the low pressure than at any other point under ordinary conditions. The steam accounted for may be computed at any point between cut-off and release on a diagram from any cylinder by the same rules and formulas used for simple engines, but in order that the area, stroke and number of revolutions may cancel as shown we must use the M. E. P. which would be equivalent in effect in the cylinder with which we are working to the aggregate of the several mean effectives in their respective cylinders.

The effect of a given mean effective pressure is proportionate to the displacement per unit of time of the cylinder in which it works. A given mean effective pressure will produce twice the power in a cylinder having twice the area, with the same piston speed. So if we wish to find how much M. E. P. would be necessary to develope an amount of power in the low pressure cylinder equivalent to that developed by a given M. E. P. in the high we must divide the M. E. P. by the ratio of the displacements between the high and low pressure cylinders. To find this ratio multiply the square of the diameter, the stroke, and the revolutions per minute of each cylinder together, and divide the product from the larger cylinder by that from the smaller. As in ordinary multi-cylinder engines all the cylinders have the same length of stroke and number of revolutions per minute, these factors cancel, and the operation is reduced to dividing the square of the diameter of the larger cylinder by the square of the diameter of the smaller, or dividing the larger by the smaller diameter and squaring the quotient.

To refer the mean effective pressure of one cylinder to another, multiply the given M. E. P. by the ratio between the cylinder displacements if the cylinder to which it is to be referred is the smaller, or divide if it is the larger. In a compound engine having cylinders 12 and 24 inches in diameter, running at the same piston speed, the diagrams show 38 pounds of M. E. P. in the high pressure and 9.18 pounds in the low. Refer the mean effective pressure to the low pressure cylinder.

The ratio between the cylinders is

$$(24 \div 12)^2 = 4$$

Then 38 pounds in the high pressure cylinder would be equalled by $38 \div 4 = 9.5$ pounds in the low pressure. Add this to the 9.18 pounds shown by the low pressure diagram and we have $9.5 + 9.18 = 18.68$ pounds of mean effective pressure which would be required to do in the low pressure cylinder alone the work of 38 in the high and 9.18 in the low. In working out the steam accounted for per horse-power per hour from the low pressure diagram therefore the M. E. P. used would be 18.68 pounds.

If we are working from the high pressure diagram we must refer the M. E. P. of the low pressure diagram to the smaller cylinder. On account of the smaller displacement, it would require four times as much pressure (4 is the ratio between the cylinder displacements) to do the work in the high pressure cylinder as in the low, so that to do the work of 9.18 pounds M. E. P. in the low pressure cylinder would require $4 \times 9.18 = 36.72$ in the high. Add to this the 38 pounds indicated by the high pressure diagram and find $36.72 + 38 = 74.72$ pounds as the M. E. P. to be used in the formula when the steam accounted for is computed from the high pressure diagram. With a triple or quadruple expansion engine proceed the same way.

With this aggregate M. E. P. proceed as though the diagram were from a single cylinder engine. When the mean effective is referred to the high pressure cylinder it is liable to become much larger than any actually obtained, and to exceed the limit of the values given in Table VI. We therefore publish Table VII taken from the Ashcroft book of instructions for the Tabor indicator a (continuation of that table), giving the values of $\frac{13,750}{\text{M.E.P.}}$ for mean effective pressures from 100 to 250 pounds,

those below 100 being found in Table VIII page 138.

If instead of making a table of $\frac{13,750}{\text{M.E.P.}}$ for various mean effective pressures we make one of 13,750 *w* for various values of *w*, we avoid using a table to find the weight per cubic foot of steam. Such a table computed by J. W. Thompson, M. E., is printed on page 146. Finding in this table the value for the pressure at the point chosen for measurement, divide it by the M. E. P. and multiply the quotient by *F*—X, or by the ratio of the horizontal line across the diagram to the total length of the diagram. When the points on the expansion and compression lines are at different heights the other process will be more convenient.

TABLE VIII VALUE OF $\frac{13750}{\text{M.E.P.}}$

QUANTITY OF STEAM ACCOUNTED FOR BY INDICATOR.

M. E. P. lbs.	13750 / M. E. P.	M. E. P. lbs.	13750 / M. E. P.	M. E. P. lbs.	13750 / M. E. P.	M. E. P. lbs.	13750 / M. E. P.	M. E. P. lbs.	13750 / M. E. P.
101.	136.1	131.	104.9	161.	85.4	191.	71.9	221.	62.2
102.	134.8	132.	104.1	162.	84.8	192.	71.6	222.	61.9
103.	133.4	133.	103.3	163.	84.3	193.	71.2	223.	61.6
104.	132.2	134.	102.6	164.	83.8	194.	70.8	224.	61.3
105.	130.9	135.	101.8	165.	83.3	195.	70.5	225.	61.1
106.	129.7	136.	101.1	166.	82.8	196.	70.1	226.	60.8
107.	128.5	137.	100.3	167.	82.3	197.	69.7	227.	60.5
108.	127.3	138.	99.6	168.	81.8	198.	69.4	228.	60.3
109.	126.1	139.	98.9	169.	81.3	199.	69.0	229.	60.0
110.	125.0	140.	98.2	170.	80.8	200.	68.7	230.	59.7
111.	123.8	141.	97.5	171.	80.4	201.	68.4	231.	59.5
112.	122.7	142.	96.8	172.	79.9	202.	68.0	232.	59.2
113.	122.6	143.	96.1	173.	79.4	203.	67.7	233.	59.0
114.	120.6	144.	95.4	174.	79.0	204.	67.4	234.	58.7
115.	119.5	145.	94.8	175.	78.5	205.	67.0	235.	58.5
116.	118.5	146.	94.1	176.	78.1	206.	66.7	236.	58.2
117.	117.5	147.	93.5	177.	77.6	207.	66.4	237.	58.0
118.	116.5	148.	92.9	178.	77.2	208.	66.1	238.	57.7
119.	115.5	149.	92.2	179.	76.8	209.	65.7	239.	57.5
120.	114.5	150.	91.6	180.	76.3	210.	65.4	240.	57.2
121.	113.6	151.	91.0	181.	75.9	211.	65.1	241.	57.0
122.	112.7	152.	90.4	182.	75.5	212.	64.8	242.	56.8
123.	111.7	153.	89.8	183.	75.1	213.	64.5	243.	56.5
124.	110.8	154.	89.2	184.	74.7	214.	64.2	244.	56.3
125.	110.0	155.	88.7	185.	74.3	215.	63.9	245.	56.1
126.	109.1	156.	88.1	186.	73.9	216.	63.6	246.	55.8
127.	108.2	157.	87.5	187.	73.5	217.	63.3	247.	55.6
128.	107.4	158.	87.0	188.	73.1	218.	63.0	248.	55.4
129.	106.5	159.	86.4	189.	72.7	219.	62.7	249.	55.2
130.	105.7	160.	85.9	190.	72.3	220.	62.5	250.	55.0

TABLE IX. Values of 13740 W.

T.P.	0	1	2	3	4	5	6	7	8	9
3	117.300	121.015	124.717	128.406	132.083	135.748	139.399	143.075	146.665	150.279
4	153.880	157.514	161.137	164.750	168.353	171.945	175.527	179.098	182.659	186.210
5	189.750	193.336	196.914	200.483	204.044	207.598	211.142	214.679	218.208	221.728
6	225.240	228.799	232.351	235.897	239.437	242.970	246.497	250.017	253.531	257.039
7	260.540	264.956	267.566	271.071	274.570	278.063	281.550	285.031	288.506	291.976
8	295.440	298.922	302.400	305.872	309.338	312.800	316.256	319.708	323.154	326.594
9	330.030	333.488	336.941	340.389	343.833	347.273	350.707	354.137	357.563	360.984
10	364.400	367.842	371.280	374.714	378.144	381.570	384.992	388.410	391.824	395.234
11	398.640	402.064	405.485	408.902	412.315	415.725	419.131	422.534	425.933	429.328
12	432.720	436.120	439.517	442.911	446.301	449.688	453.071	456.451	459.828	463.200
13	466.570	469.950	473.326	476.699	480.068	483.435	486.798	490.159	493.516	496.869
14	500.220	503.596	506.968	510.338	513.706	517.070	520.432	523.790	527.146	530.500
15	533.850	537.213	540.573	543.930	547.285	550.638	553.987	557.334	560.679	564.011
16	567.360	570.713	574.063	577.411	580.757	584.100	587.441	590.780	594.115	597.449
17	600.780	604.109	607.435	610.759	614.081	617.400	620.717	624.031	627.343	630.653
18	633.960	637.265	640.567	643.867	647.165	650.460	653.353	657.043	660.331	663.617
19	666.900	670.200	673.498	676.793	680.086	683.378	686.666	689.953	693.238	696.520
20	699.800	703.098	706.394	709.688	712.980	716.279	719.558	722.844	726.128	729.410
21	732.690	735.968	739.244	742.518	745.790	749.060	752.328	755.594	758.858	762.120
22	765.380	768.660	771.938	775.215	778.490	781.763	785.034	788.303	791.570	794.836
23	798.100	801.362	804.622	807.881	811.138	814.393	817.646	820.897	824.146	827.334
24	830.640	833.908	837.175	840.440	843.703	846.965	850.225	853.484	855.741	859.995
25	863.250	866.502	869.753	873.002	876.249	879.495	882.739	885.982	889.223	892.462
26	895.700	898.936	902.171	905.404	908.635	911.865	915.093	918.320	921.545	924.768
27	927.990	931.210	934.429	937.646	940.831	944.075	947.287	950.498	953.707	956.914
28	960.120	963.352	966.583	969.813	973.041	976.268	979.493	982.217	985.939	989.160
29	992.380	995.598	998.815	1002.031	1005.245	1008.458	1011.669	1014.879	1018.087	1021.294
30	1024.500	1027.701	1030.907	1034.109	1037.309	1040.508	1043.705	1046.901	1050.095	1053.288
31	1056.840	1059.670	1062.859	1066.047	1069.233	1072.418	1075.601	1078.783	1081.963	1085.142

TABLE IX. Values of 13750 W. (Continued.)

T.P.	0	1	2	3	4	5	6	7	8	9
32	1088.320	1091.528	1094.736	1097.942	1101.146	1104.350	1107 552	1110.754	1113.954	1117.152
33	1120.250	1123.546	1126.742	1129.936	1133.128	1136.420	1139.510	1142.700	1145.888	1149.074
34	1152.260	1155.444	1158.628	1161.810	1164.990	1168.170	1171.348	1174.526	1177.702	1180.876
35	1184.050	1187.222	1190.394	1193.564	1196.732	1199.900	1203.066	1206.232	1209.396	1212.558
36	1215.720	1218.917	1222.112	1225.307	1228.500	1231.693	1234.884	1238.075	1241.264	1244.453
37	1147.640	1250.827	1254.012	1257.197	1260.380	1263.563	1266.744	1269.925	1273.104	1276.283
38	1279.460	1282.637	1285.812	1288.987	1292.160	1295.333	1298.504	1301.675	1304.844	1308.013
39	1311.180	1314.347	1317.512	1320.677	1323.840	1327.003	1330.164	1333.325	1336.484	1339.643
40	1342.800	1345.957	1349.112	1352.267	1355.420	1358.573	1371.724	1364.875	1368.024	1371.173
41	1374.320	1377.467	1380.612	1383.757	1386.900	1390.043	1393.184	1396.325	1399.464	1402.603
42	1405.740	1408.877	1412.012	1415.147	1418.280	1421.413	1424.544	1427.675	1430.804	1433.933
43	1437.060	1440.230	1443.398	1446.566	1449.734	1452.900	1456.066	1459.230	1462.394	1465.558
44	1468.720	1471.882	1475.042	1478.202	1481.362	1484.520	1487.678	1490.834	1493.990	1497.146
45	1500.300	1503.454	1506.606	1509.758	1512.910	1516.060	1519.210	1522.359	1525.506	1528.654
46	1531.800	1534.946	1538.090	1541.234	1544.378	1547.520	1550.662	1553.802	1556.942	1560.082
47	1563.220	1566.358	1569.494	1572.630	1575.766	1578 900	1582.034	1585.166	1588.298	1591.430
48	1594.560	1579.690	1600.818	1603.946	1607.074	1610.200	1613.326	1616.450	1619.574	1622.698
49	1625.820	1628.942	1632.062	1635.182	1638.302	1641.420	1644.538	1647.654	1650.770	1653.886
50	1657.000	1660.114	1663.266	1666.338	1669.450	1672.560	1675.670	1678.778	1681.886	1684.994
51	1688.100	1691 206	1694.310	1697.414	1700.518	1703.620	1706.722	1709 822	1712.922	1716.022
52	1719.120	1722.218	1725.314	1728.410	1731.506	1734.600	1737.604	1740.786	1743.878	1746.970
53	1750.060	1753.150	1756.238	1759.327	1762.414	1765.500	1768.586	1771.670	1774.754	1777.838
54	1780.920	1784.002	1787.082	1790.162	1793.242	1796.320	1799.398	1802.474	1805.550	1808.626
55	1811.700	1814.829	1817.957	1821.084	1824.211	1827.338	1830.463	1833.588	1836.713	1839.837
56	1842.960	1846.083	1849.205	1852.326	1855.447	1858.568	1861.687	1864.806	1867.925	1871.043
57	1874.160	1877.277	1880.393	1883.508	1886.623	1889.738	1892.851	1895.964	1899.077	1902.189
58	1905.300	1908.411	1911.521	1914.630	1917.739	1920.848	1923.955	1927.062	1930.169	1933.275
59	1936.380	1939.485	1942.589	1945.692	1948.795	1951.898	1954.999	1958 100	1961.201	1964.301
60	1967.400	1970.499	1973.597	1976.694	1979.791	1982.888	1985.983	1989.078	1992.173	1995.267

CHAPTER XVI.

DIAGRAMS FROM COMPOUND ENGINES, CLEARANCE NEGLECTED

So far as taking the diagrams from a compound engine, figuring the horse-power, the water accounted for, etc., the directions already given will suffice. The diagram will be taken just as though the cylinder operated upon were the only one concerned, the selection of the spring being governed by the range of pressures in that cylinder and convenience in reducing the diagrams to a common scale, as will be explained. The indicated horse-power is found by computing the horse-power of each cylinder in the ordinary manner from its own diagrams and adding the indicated horse-power of the several cylinders for the total power of the engine. The steam accounted for per horse-power per hour is obtained by referring the mean effective pressures of the several cylinders to the cylinder in which the pressure used for the computation is measured, as explained in the chapter on steam consumption from the diagram. Each diagram is a representation of the distribution and use of steam in the conditions of its own cylinder, and may be studied in connection with a theoretical diagram for those conditions, just as a diagram from a single cylinder engine would.

In order to study the action of the steam in the engine as a whole, however, and to compare it with an ideal expansion of steam through the range adopted, we must study the diagrams in their relation to one another, and this involves their reconstruction in several particulars. In the first place, to be comparable, the diagrams must be upon the same scale. For the high pressures used in the initial cylinder of compound engines we must use a stiff spring. In order to get a large card on the low pressure cylinder we use a spring of lower scale. When we wish

to compare the resulting diagrams we must reduce them to the same scale, and as we can work more accurately upon a large than a small scale, it is preferable to increase the height of the high pressure diagram to that which it would have been if taken with the same spring as the other.

Suppose we have a compound engine with the low pressure cy-

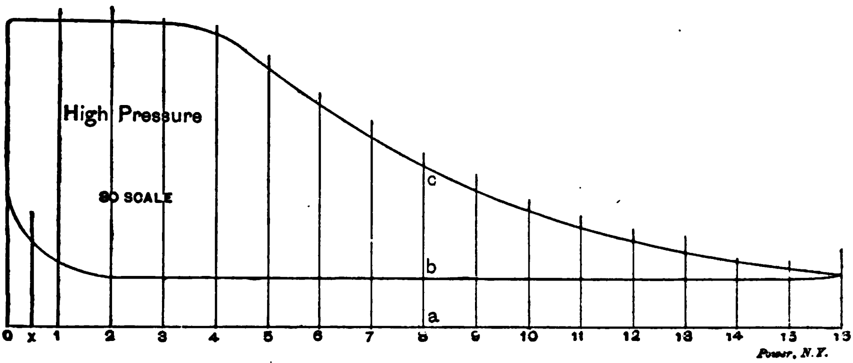


FIG. 115

linder twice the diameter of the high, cutting off at a quarter stroke in both cylinders, with a boiler pressure of 160 pounds absolute and 26 inches of vacuum, the stroke of both cylinders being equal; and that from this engine we had got the diagrams Fig. 115 with an 80 scale, and Fig. 116 with a 20 scale. Neglecting for the present the influence of clearance, let us combine them so as to show the continuous action of the steam in the whole engine.

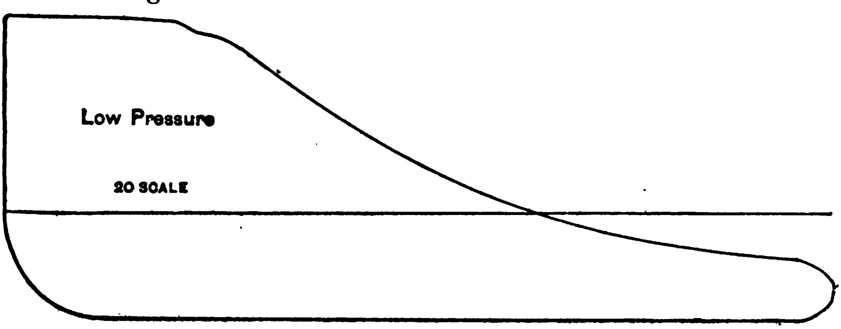


FIG. 116

If the high pressure diagram had been taken with a 20 instead of an 80 spring every point upon it would have been $\frac{80}{20} = 4$ times as high above the atmospheric line as the diagram shows it. The first step, therefore, is to redraw this diagram four times its present height.

Divide the diagram into a convenient number of equal parts and erect ordinates upon the divisions. In Fig. 115 sixteen spaces have been used, as they are easily obtained by successive halvings. Measure the distances from the atmospheric line to the forward and back pressure lines of the diagram on each

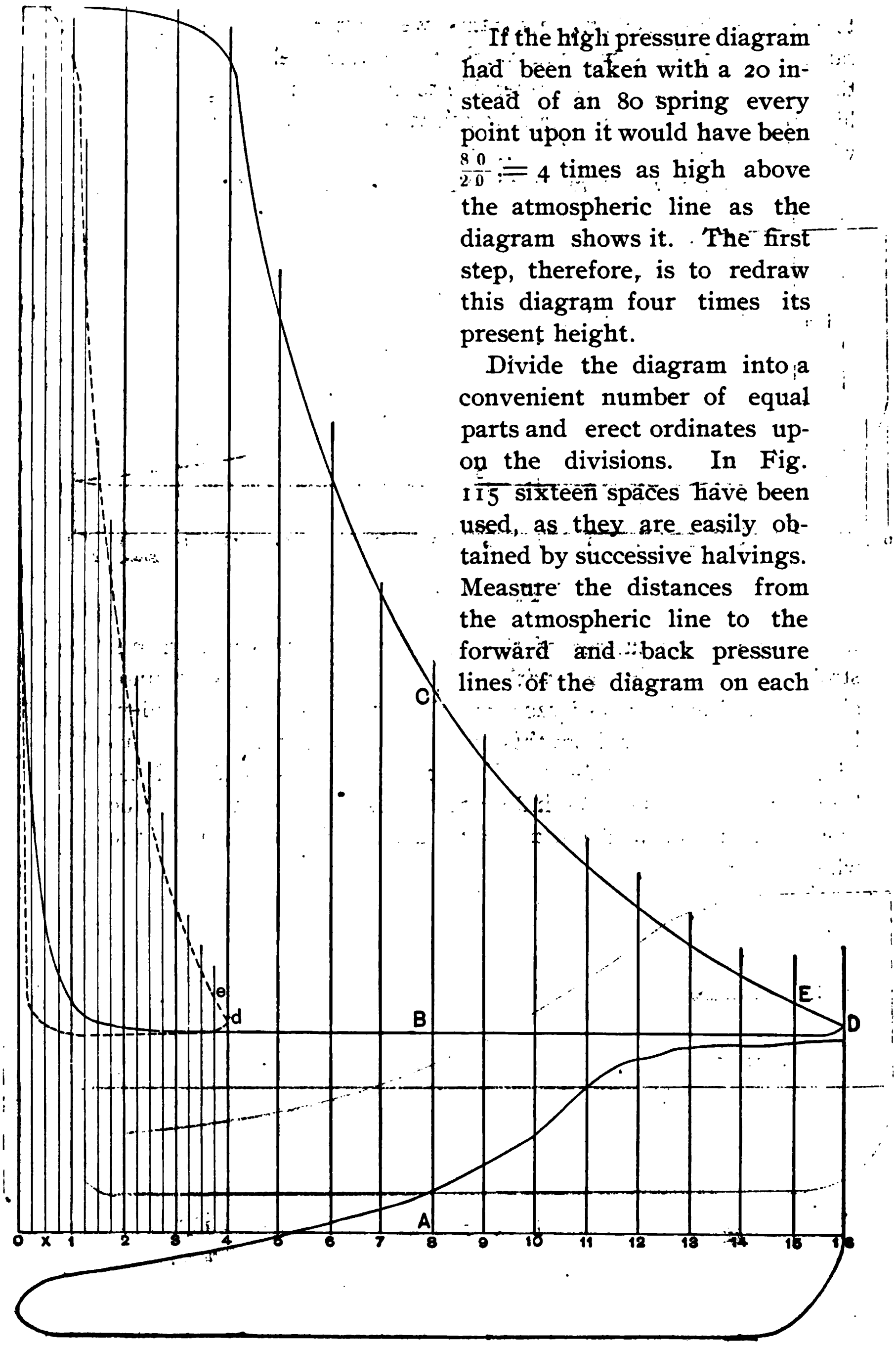


Fig. 117.

ordinate, and transfer these distances multiplied by four to the corresponding ordinate upon the larger diagram. On ordinate 8, for example, the distances *A B* and *A C* in Fig. 117 are four times the distances *a b* and *a c* on the corresponding ordinate in Fig. 115. A pair of proportional dividers will be found convenient for this work. Drawing a line through the points thus indicated, we obtain the diagram shown in Fig 117. Where sudden changes of pressure occur, so that it would be difficult to draw the line correctly between points so far apart, additional ordinates may be put in, as at *x*, Fig. 115, putting an ordinate in the same position on the reconstructed diagram.

We can now consider the diagrams somewhat in their relation one to another by placing them together, as shown in Fig. 117, where the low pressure diagram is just as it was drawn by the indicator. The steam is expanded to about 40 pounds, exhausts into the receiver, and the space between the back pressure line of the high pressure diagram and the steam line of the low pressure shows the loss in going through the ports and receiver between the two cylinders.

But even now we are not able to compare the diagrams with a theoretical diagram showing the expansion of the steam from the initial pressure to the terminal in the low pressure cylinder. To do this we must reduce the diagrams to the same scale of volumes. If the area of the high pressure piston was one square foot, then every foot of movement of that piston would expand the steam behind it one cubic foot. If the low pressure piston has twice the diameter of the high it would have four times the area, and each foot of movement of the low pressure piston would add four cubic feet to the volume of the steam. One foot of movement of the low pressure piston is equal, then, to four feet of the high, and since the movement of the piston is represented by the length of the diagram, the high pressure diagram, to be comparable to the low, should be only one-fourth the length of the low pressure diagram.

This calculation has been made on the assumption that the larger cylinder had twice the diameter of the smaller and that the strokes were equal. In general the diagrams should be to each other in length as the volumes of their respective cylinders. The volume of the cylinder (clearance neglected) is the cross-sectional

area multiplied by the length of the stroke; the area is the square of the diameter multiplied by .7854. Then letting

d = diameter high pressure cylinder.
D = " low " "
l = length stroke high pressure cylinder.
L = " " low " "

the ratio of the lengths of the diagrams would be

$$\frac{d^2 \times .7854 \times l}{D^2 \times .7854 \times L}$$

The decimals cancel, and as the stroke is ordinarily the same in both cylinders the lengths usually cancel also, so that usually the ratio of the diagram lengths is

$$\frac{d^2}{D^2}$$

In our case we found this ratio to be ¼, that is, the high pressure diagram must be ¼th as long as the low.

Lay off on the admission end of the enlarged diagram, Fig. 117, a length equal to ¼th the length of the low pressure diagram, divide it into as many spaces as the original diagram was at first divided, 16 in this case, and erect ordinates as shown. Then transfer the pressures on the ordinates of the large diagram to the corresponding ordinates of what will be the shortened diagram. For instance, we made a dot d on the last ordinate of the shortened diagram at the same height as the point D, where the line touches the last ordinate of the large diagram;

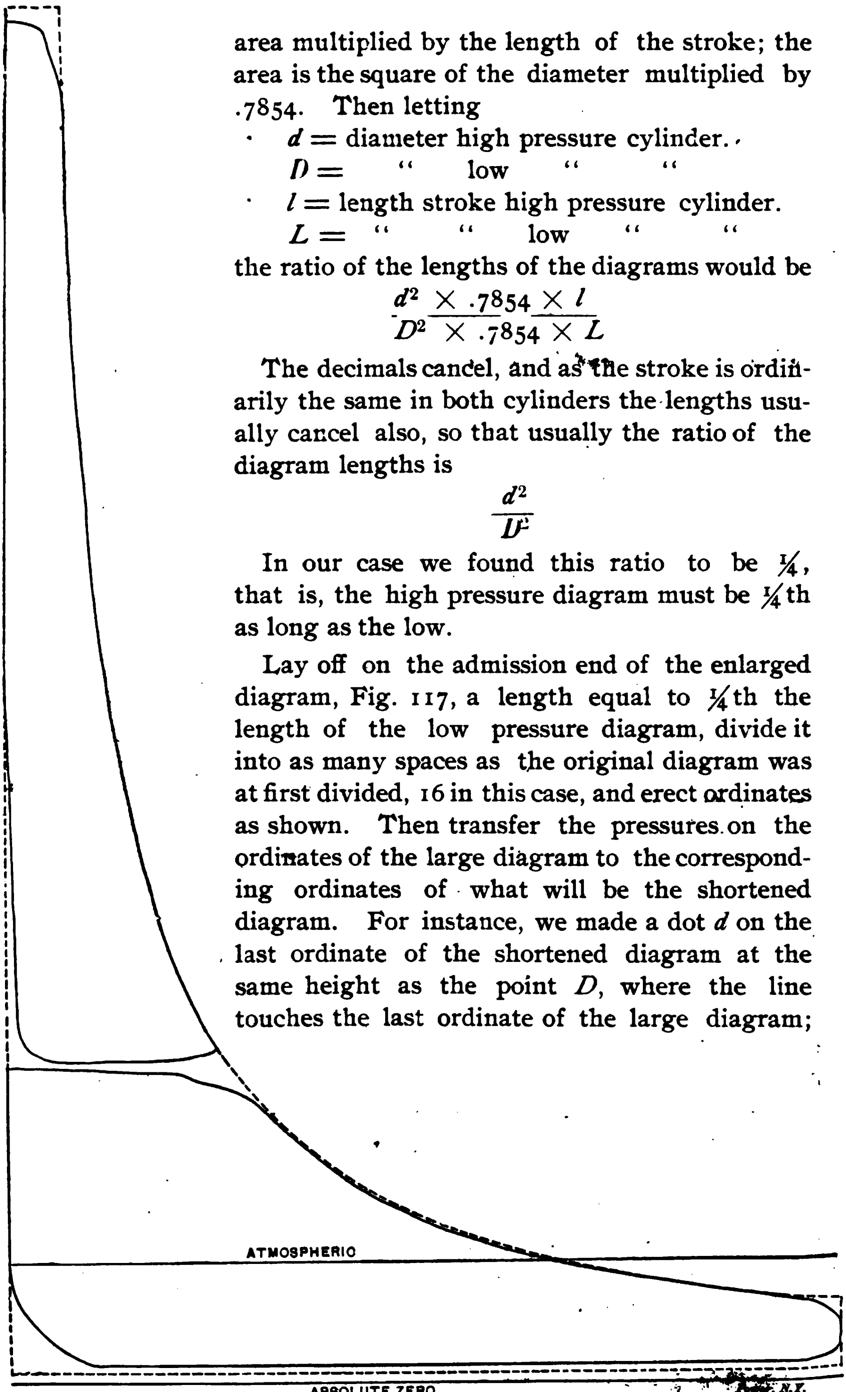


Fig. 118.

another at *e* on the second ordinate; counting from the right, at the same height as *E* on the corresponding ordinate of the large diagram; and so on for both the forward and back pressure lines upon all the sixteen ordinates. Connecting these points we get the diagram shown by the dotted line, as though it had been taken with a 20 spring and only one-quarter the movement to the paper barrel that the low pressure diagram had. If this diagram is placed above the low pressure diagram, as in Fig. 118, we have a representation of the continuous action of the steam and can draw about it the theoretical diagram, as shown by the dotted line, showing how much of the enclosed area is covered by the diagrams from the engine, and how nearly perfect the utilization of the steam has been.

CHAPTER XVII.

DIAGRAMS FROM COMPOUND ENGINES, CLEARANCE CONSIDERED.

In the last chapter we described the combination of diagrams from the various cylinders of a compound or triple expansion engine so as to be comparable with an equivalent action of the steam in a single cylinder. We neglected clearance for the sake of simplicity, but it now becomes necessary to proceed to the consideration of the clearance in such a combination. Its treatment is shown in Fig 119 for a two cylinder engine in which the diameters of the cylinders are as 2 to 1, making the volumes for equal strokes as 4 to 1. In both cylinders the clearance is $\frac{1}{12}$ or $8\frac{1}{2}$ per cent of the displacement. Draw the line of zero pressure, perfect vacuum, *O X* Fig. 119, and at a distance *a b* (= $\frac{1}{12}$th the length of the low pressure diagram) from the admission line erect the line *O A* of zero volume. Then set the

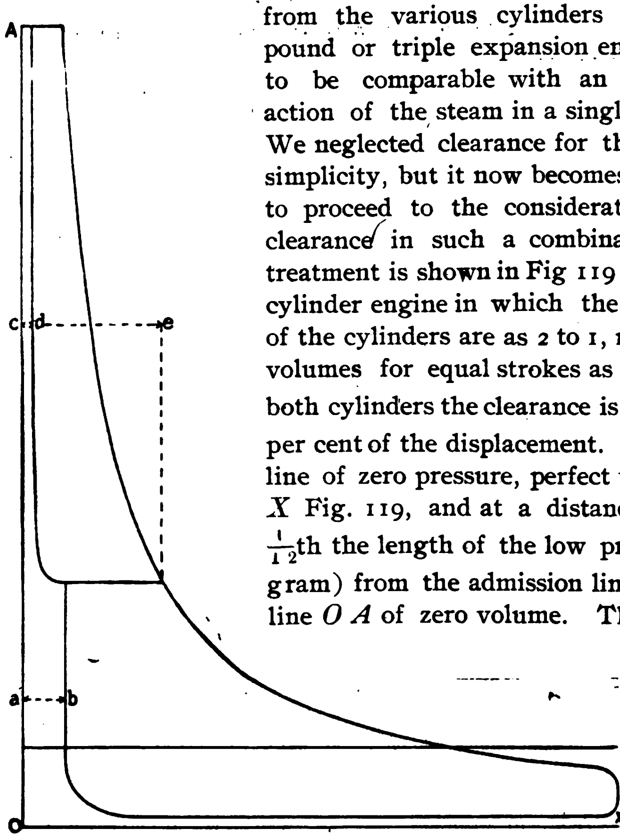


Fig. 119.

reconstructed high pressure diagram at such a distance from the line $O A$ that the clearance space $c d$ shall be the proper percentage (in this case $\frac{1}{12}$ or $8\frac{1}{2}$ per cent) of the length $c e$ of that diagram. In other words, add the clearance line in the usual manner to the reconstructed diagrams, and in combining make the clearance lines coincide. Let us consider a little further the action of steam in compound engines, using for the purpose conventional or theorectical diagrams drawn upon the same scales for both cylinders. Let us take first the engine with no receiver but with the high pressure exhausting directly into the low, and the pis-

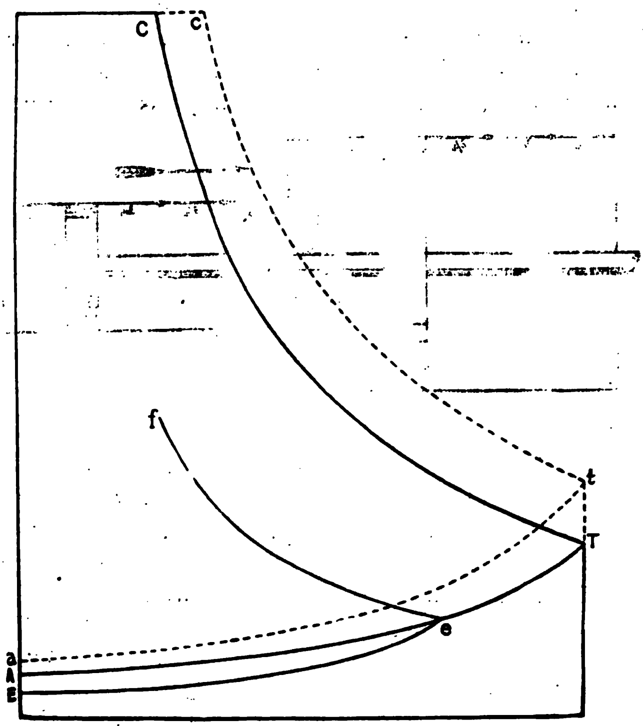


Fig. 120.

tons moving together, as in a tandem, or with equal opposite movements, as with a cross-compound the cranks of which are opposite. Suppose the cut-off to take place in the high pressure cylinder at one quarter stroke, C, Fig. 120, in which case the

steam would be expanded to the terminal pressure *T*, say 30 pounds. Now suppose a valve as at *A*, Fig. 121, between the two cylinders, to open, and the pistons to commence to move toward the left. As the area of the low pressure cylinder is four times as great as that of the high, every inch of movement will add four times the volume in the low pressure cylincer that is taken up by the forward movement of the high pressure piston. When, for instance, the pistons have made one-quarter of their stroke, and are in the position shown, the steam will still have three-quarters as much room to occupy in the high pressure cylinder as it had before the return stroke was commenced, and in addition it will have one-quarter of the low presssure cylinder. As the low pressure has four times the volume of the high, the steam will have in one-quarter of the low pressure as much room

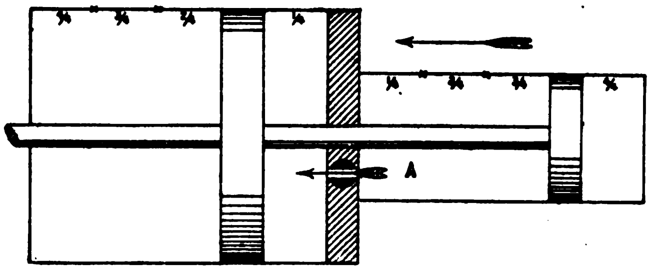


Fig. 121.

as it had in the high pressure cylinder at the end of the forward stroke, besides the three-quarters of its original volume, still left in the high pressure cylinder. Its volume has, therefore, at the point under consideration, been expanded to 1¾ that at the termination of the forward stroke, and knowing that the pressure is inversely as the volume (see chapter on expansion line,) we divide the terminal pressure 30, by 1¾, and find a little over 17 pounds as the pressure at the point e, Fig. 120. Locating the pressure at the other points in the same manner, we find that the back pressure on the high pressure piston, which in this case would also be the forward pressure on the low pressure, would follow the line *T A*, Fig. 120 with an uninterrupted passage of the steam between the cylinders throughout the stroke.

If the point of cut-off in the high pressure cylinder were to change, it would change the terminal pressure *T* in that cylinder,

and correspondingly increase or diminish the initial pressure in the low. Instead of cutting off at *C*, Fig. 120, one-quarter of the stroke, we cut off at *c*, one-third of the stroke, the terminal pressure would be t instead of *T*, and the back pressure line of the high pressure diagram, which is at the same time the steam line of the low pressure diagram, would be *t a*. If, on the other hand, the cut-off is earlier in the high pressure, the initial for the low pressure will be lowered and less work will be done in that cylinder.

Now suppose that instead of remaining open, the valve *A*, Fig. 121, between the cylinders, closed at quarter stroke, giving us a one-quarter cut-off in the low pressure cylinder as well as in the high. This would carry the expansion line of the low pressure along the line *e E*, but it would shut up the exhaust of the high pressure cylinder, and compression would commence at *e*, running the back pressure line rapidly up in the direction *e f*.

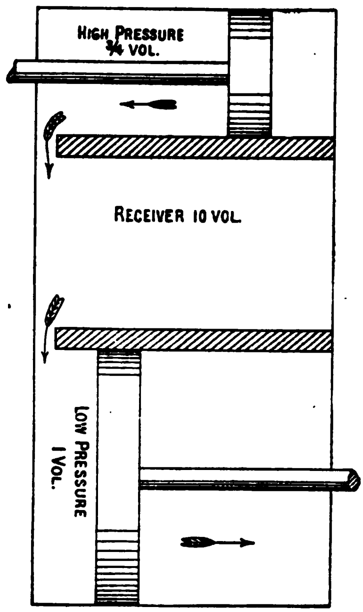


Fig. 122.

Now suppose that instead of exhausting directly into the low pressure cylinder the high pressure exhausts into a receiver or reservoir, from which the low pressure takes its supply, as in Fig. 122.

This receiver can be so large in proportion to the cylinders that the fluctuations in the quantity of steam taken from and delivered to it during the stroke will affect the pressure but little. Understand that the low pressure cylinder must take out of the receiver as much steam as the high pressure delivers to it. It is obvious that it cannot continuously take out more and if it does not take out as much the steam would accumulate in the receiver and raise the pressure until the volume taken by the low pres-

sure contained as much steam as the high pressure was delivering. Suppose the capacity of the receiver to be 10 times that of the high pressure cylinder. At the beginning of the stroke we shall have one volume in the high pressure cylinder and ten volumes in the receiver of steam at the terminal pressure $T = 30$ pounds, 11 volumes in all. At quarter stroke, Fig. 122 we shall have three-quarters of a volume in the high pressure, ten volumes in the receiver, and one volume in the low, one-quarter of

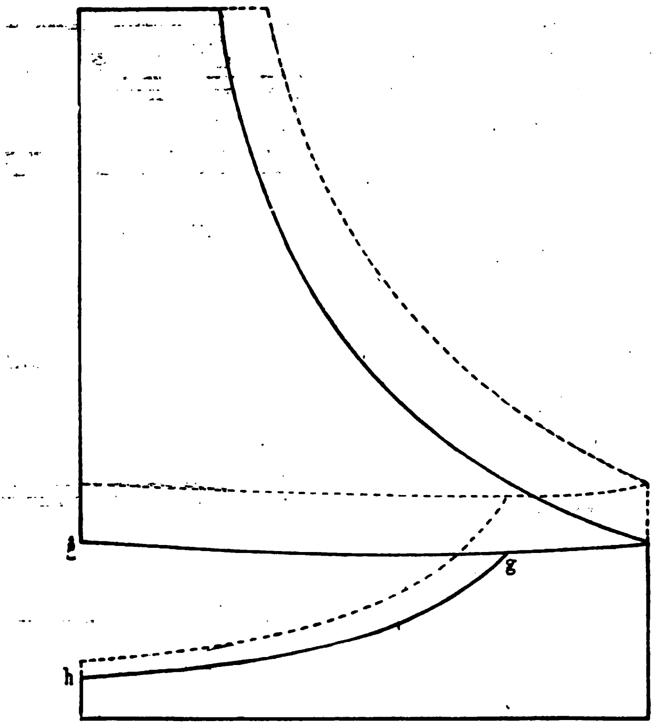


Fig. 123.

the low pressure cylinder being equal to the whole volume of the high, 11¾ volumes in all. The pressure will have fallen then to only $\frac{11}{11.75}$ of the original 30, or to a little over 28 pounds, as at *g*, Fig. 123, instead of to 17, as at *e*, Fig. 120. Suppose now the valve *A*, Fig. 121, to close, *i. e.*, cut-off to occur on the low pressure cylinder. The expansion in that cylinder would

follow the line $g\ h$, Fig. 123, while the high pressure cylinder would continue to exhaust into the receiver, and at the end of the stroke would have taken back that excess of three-quarters of a volume which it had when cut-off occurred on the low pres-pressure, and brought the pressure back from 28 to 30 pounds, the counter-pressure following the line $g\ i$.

Suppose a heavier load to come on the engine, changing the point of cut-off from one-quarter to one-third stroke. First let

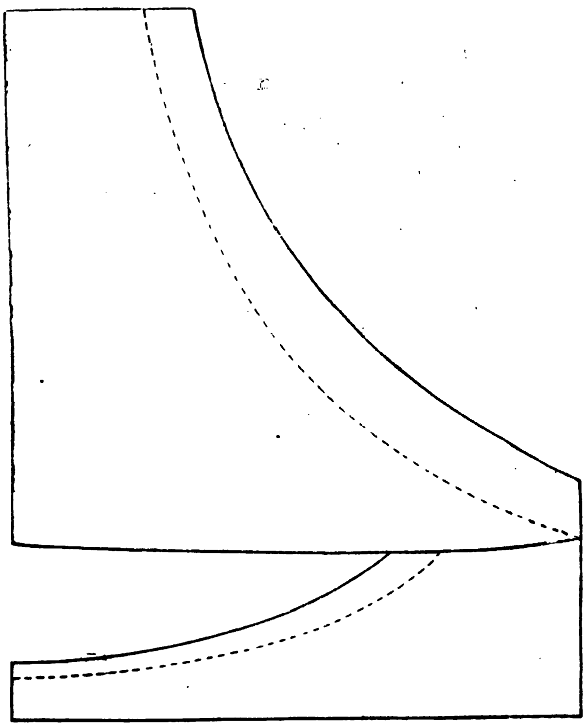


Fig. 124.

us consider the effect with a fixed cut-off on the low pressure cylinder, which we will allow to remain at one-quarter stroke. The result is shown by the dotted diagram in Fig. 123. The greater portion of the increase of load is taken by the low pressure cylinder, on which the cut-off has not changed, the area gained by the later cut-off in the high pressure cylinder being largely offset by the loss of area due to the increase of back pres-

sure through the higher terminal. Notice also that with the low pressure cut-off set at one-quarter, the volume which the low pressure cylinder takes out of the receiver each stroke just equals the volume delivered to it by the high pressure, so that whatever the terminal pressure, the high pressure diagram will end in a point.

Suppose now there had been an automatic cut-off on both cylinders, and that the low pressure cut-off changed to one-third stroke too. The low pressure cylinder has four times the volume of the high. One-third of the low would have $\frac{1}{3} \times 4 = 1\frac{1}{3}$ times the volume of the high, so that for every cubic foot of steam that the high pressure cylinder delivers to the receiver the low pressure cylinder takes out $1\frac{1}{3}$ cubic feet. Since there is a greater volume going out of the receiver than there is going into it, the pressure will fall until the greater volume taken out by the low pressure cylinder contains only the same quantity or weight of steam as that delivered in a smaller volume by the high pressure cylinder. In other words, the receiver pressure will fall until the cylinderful of steam delivered to the receiver at 40 pounds will expand to $1\frac{1}{3}$ times its volume in the receiver which should require a receiver pressure of $40 \div 1\frac{1}{3} = .30$ pounds. We should therefore have a diagram like Fig. 124, where the dotted lines represent both cylinders cutting off at one-quarter stroke, the full lines, both cylinders cutting off at one-third stroke.

CHAPTER XVIII.

ERRORS IN THE DIAGRAM.

In treating of the reducing motion we have described in kind the various errors to which it is liable. It now remains to consider them in degree. Fig. 125 shows the error which would result from taking the motion from a pin on a lever like Fig. 126, vibrating through about 90 degrees. A diagram which

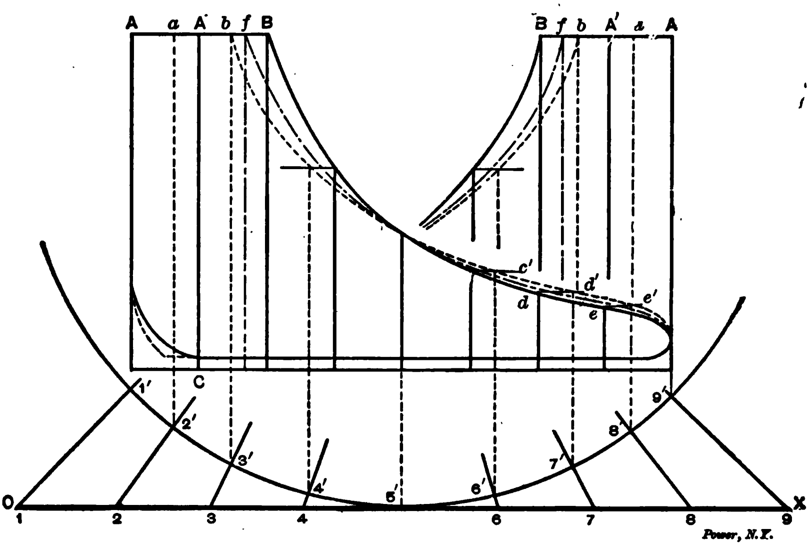


Fig. 125.

should follow the full line would be distorted by this arrangement to that shown by the dotted lines. The cut-off would appear too early, the expansion line would hold up too much for the apparent cut-off, but would be below its proper position in the first of the stroke, crossing the correct line at the center, and

making the terminal appear higher than it should be. It makes the release and compression appear late and reduces the area of the diagram, and hence the apparent indicated horse-power. Both the right and left handed diagrams, *i. e.*, those from the head and the crank end, are affected the same way. When you see a diagram which resembles the dotted one in Fig. 125, look over the reducing motion.

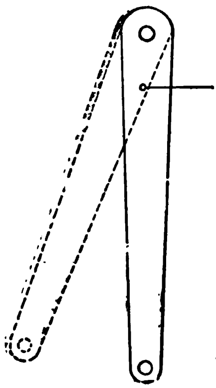


Fig. 126.

As just stated, Fig. 125 was drawn upon the assumption that the lever vibrated through 90 degrees. This is excessive. It is recommended to use a lever not less than one and a half times the length of the stroke. This gives a vibration between 35 and 40 degrees. In Fig. 127 is shown the distortion due to using a lever like Fig. 126, one and a half times the length of the stroke, taking the motion from a pin in the lever, and a cord led off parallel to the guides.

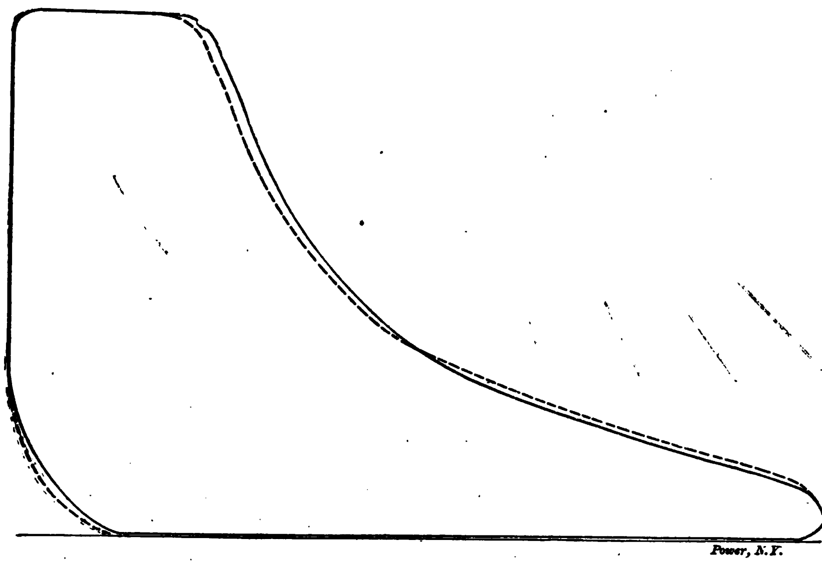


Fig. 127.

The distortion is much less than with the shorter lever, and the purpose for which the diagram is taken must determine

whether this amount can be tolerated for the sake of simplicity in the reducing motion. When we measure for a carpet we do not take into account the fractions of an inch, and when we weigh coal we do not pay attention to the ounces. In ordinary indicating to see that the valve gear has not become deranged, to make a rough cast of the power for purposes of record, etc., we need not be so precise as though we were testing a cruiser, when the difference of one pound mean effective pressure would mean ten thousand dollars to the builders; or a steam plant where a few horse-power more or less would determine for or against the guarantee; or when with Hirn, we undertake to trace from the diagram the distribution and disposition of the heat units going through the plant. This is when the indicator and its user must get right down to extreme accuracy, and after every precaution is used the results will still be too far from the truth. This motion cannot be corrected by the use of a brumbo pulley, for the pulley would not move through equal arcs for equal

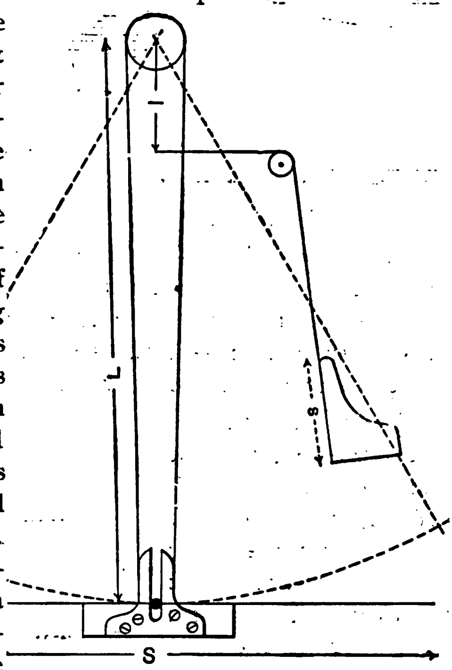


Fig. 128.

movements of the cross-head. It would pull the cylinder a distance equal to 4′, 5′, Fig. 125, in the middle of the stroke, and only that equal to 1′, 2′, etc., at the ends, so that instead of being equally divided for equal movements of the piston the diagram would be divided irregularly, as are the spaces on the arc. If this arc were straightened out, reduced to the length of the diagram without disturbing the proportion of the spacing, corresponding ordinates, as 3′ *f*, erected, and the pressure transferred to these from the proper ordinates, as from *B* to *f*, we should get

the diagram represented by the broken line, showing that the use of the arc is productive of greater accuracy in this case. With a lever of constant length, as in Fig. 128, however, the use of the arc introduces an error. (See chapter on reducing motions.)

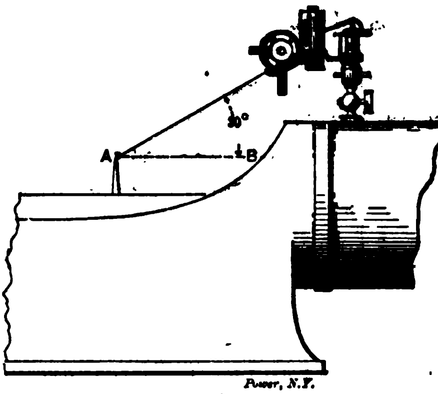


Fig. 130.

Leading the cord away from the reducing motion in any other direction than parallel with the guides introduces an error. Let us see how much. Suppose we have a pantograph, as in Fig. 129, or a reducing wheel, as in Fig. 130, and that instead of leading the cord off in the direction *A. B.* parallel with the guides, we led it off in the direction shown, the angle being 30 degrees when the cross-head is nearest to the cylinder. The resulting distortion of the diagram will be that shown in Fig. 131. When the piston has traveled one-eighth of its stroke the pencil, which should be at *A*, will be *a*, and so on for the other ordinates. Notice that this makes the apparent cut-off earlier on the head end and later on the crank end. At all times and in both directions the travel of the paper-drum is less than it should be, although it looks to be more when traveling to the right.

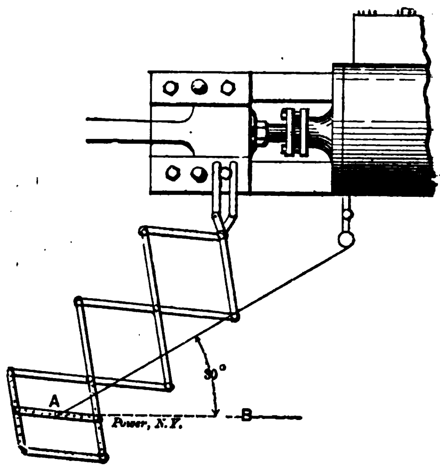


Fig. 129.

Thus, starting with *O* at the right the pin on the pantograph, when the engine cuts off at quarter stroke, will have moved a distance equal to *O* 2, but the movement of the paper-drum will

be equal to *O C* only. When the stroke is completed the pantograph pin has traveled through a distance equal to *O* 8, but the paper-drum has traveled through *O D*, the comparative movement of the pantograph pin and the paper-drum for successive eighths of the stroke being shown by the bold faced figures 1, 2, 3, etc., and the dotted ordinates to the right of them. The full line ordinates are placed upon the equal eighths of the shortened diagram *O D*. Starting at *D* backward the pantograph pin would move in the first eighth of the stroke to 1, in the second eighth to 2, etc. The corresponding position of the pencil on the paper

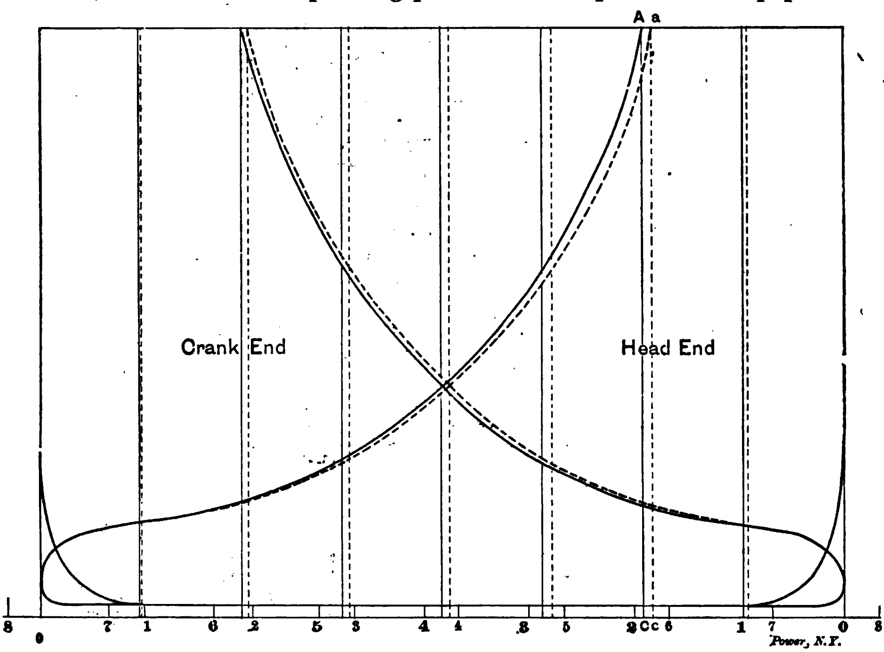


Fig. 131.

would be at the dotted ordinates as before, a less distance, it will be seen, than the actual movement in every case; but when we come to erect the full line ordinates on the even eighths of the shortened diagram they fall behind the dotted lines, showing how we can get an apparently excessive movement on the crank end with a movement really less than it should be. Notice that the distortion due to this cause tends to throw the card out of bal-

ance, affecting the diagrams from the head and crank ends in different directions, not in the same way as did the distortion of the lever motion in Fig. 125.

Another source of error in the diagram, briefly referred to on page 32, is that due to a long and indirect passage from the cylinder to the indicator. The errors introduced are: less realized pressure, lower compression and higher terminal. This subject has been discussed in the various technical papers, and varying opinions have been elicited. In order to determine this question, the author, in connection with Mr. A. C. Lippincott, undertook the tests resulting in the diagrams shown in Figs. 132-141. We designed the apparatus shown in Fig. 132.

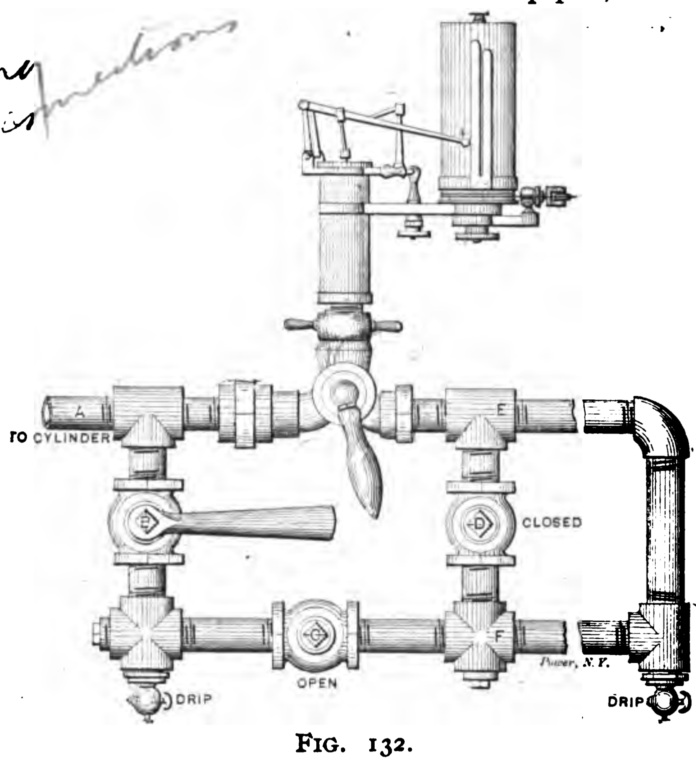


FIG. 132.

Our first test was made on an 11 by 11 Ball & Wood engine at the Roosevelt Building, New York, through the courtesy of Mr. Thomas Murphy, the engineer in charge. The engine was run-

ning at 270 revolutions per minute, driving a dynamo with a very constant load, so constant that when the pencil was held on for 20 revolutions the line of the diagram was scarcely thickened. Three and a-half feet of half-inch pipe connected the cross *F*

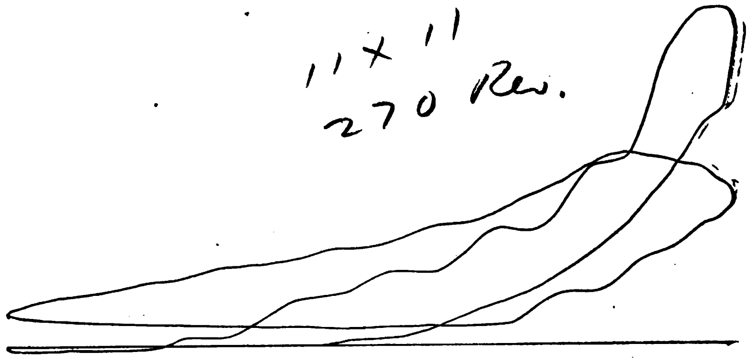


Fig. 133.

with the tee *G*, and a similar length was used between *E* and *H*, the right and left nipple *I* being about 7 inches long. This pipe was thoroughly heated and drained before each card was taken, by turning the three-way cock, so that steam could issue through the little escape orifice, opening the drips and the cock *B*, the engine running continuously.

Having taken a diagram with the direct connection, the

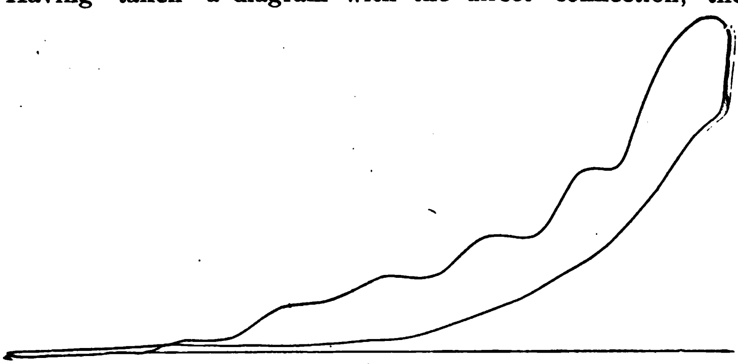


Fig. 134.

three-way cock was reversed and the cock *B* opened, compelling the steam to travel through the loop of about 8 feet of ½ inch pipe and fittings to the indicator. The result is

shown in Fig. 133. The pencil was allowed to pass over the card 20 revolutions as before, to insure that the diagram was not erratic or exceptional. This experiment was repeated over and

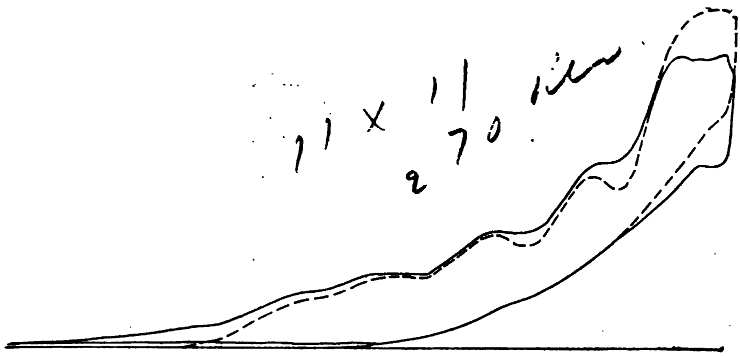


Fig. 135.

over again. Whenever we switched to the direct connection we got Fig. 134, whenever with the direct connection we opened the connection to the piping, we got Fig. 135; and when the steam

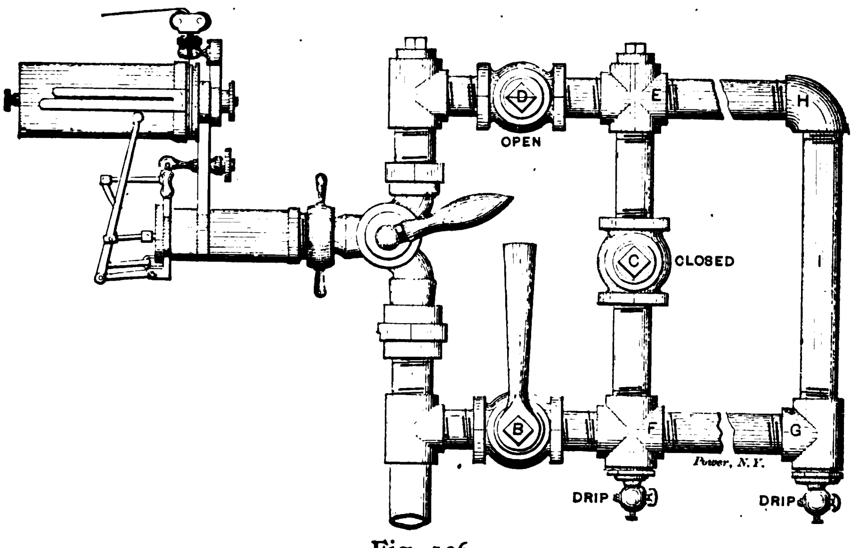


Fig. 136.

was compelled to pass around to the further side of the three-way cock to get to the indicator we got Fig. 133. The passages through the pipes and fittings were perfectly clear, and ordinary

½-inch plug cocks, half-inch fittings and the three-way cock regularly supplied with the indicator were used. The nipple *A* is screwed into the hole in the cylinder ordinarily provided for the indicator cock. When the handle of the three-

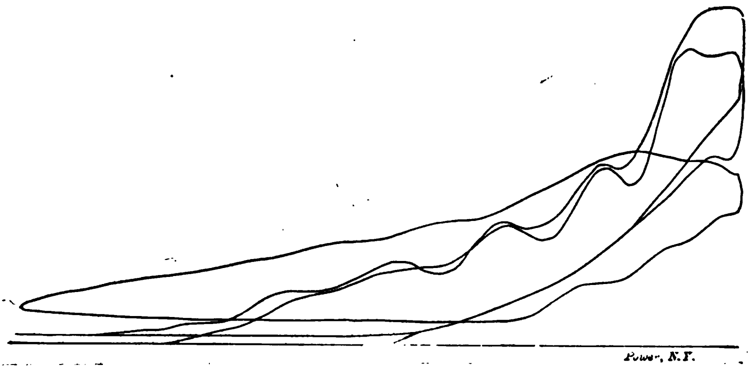


Fig. 137.

way cock is thrown to the right, as in the drawing, the steam enters the cock from the left and has a direct passage to the indicator, and if the plug cock *B* is closed the steam has no access to the extraneous piping, and the indicator is about as directly connected as it would be with the usual nipple elbow and single cock. The plug cock *C* is open and *D* is closed, so

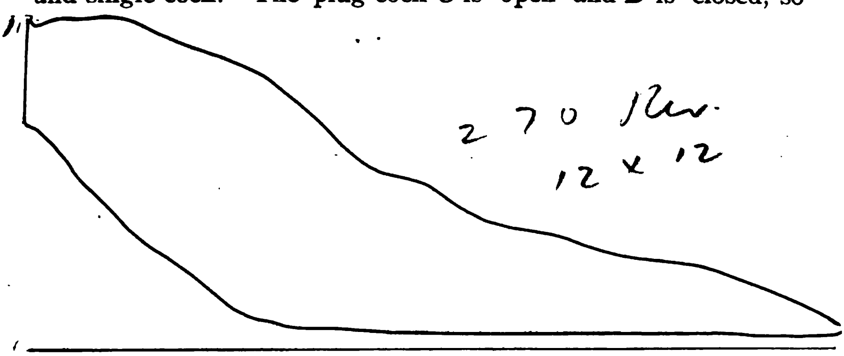
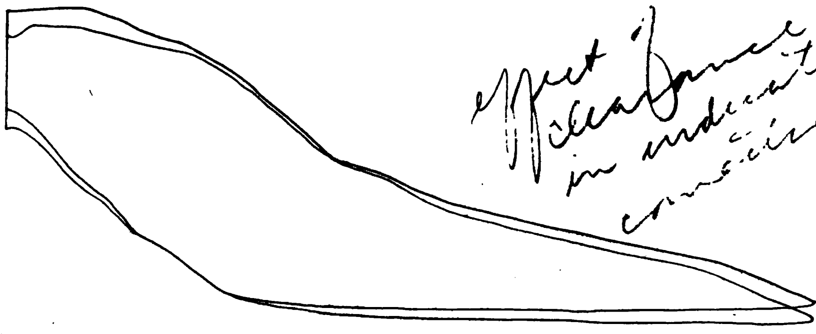


Fig. 138.

that when *B* is opened steam can pass clear around the loop and enter the three-way cock at the right, as it must do to get to the indicator when the handle of the three-way cock is swung the other way. Any sort of a circuit of piping, steam hose, or fittings

may be connected at E F for the steam to pass through on its way to the indicator. The handle of the three-way cock can also be left so as to give the steam a direct passage to the indicator and the cock B left open so as to obtain the effect of the addition to the clearance without the friction of the pipe.



Effect of clearance in indicator connection

Fig. 139.

Fig. 132 shows the apparatus as applied to a cylinder tapped at the side as are engines of the Corliss type. For engines tapped on top of the cylinder it is turned as shown in Fig. 136, which will explain the necessity of the cocks C and D.

Fig. 137 is a card on which all three diagrams were taken as

Fig. 140.

quickly as the cocks could be shifted. Through the kindness of Mr. Gillespie, in charge of the steam plant of the Young Women's Christian Association Building, we were able to repeat the experiment on a 12 by 12 New York Safety engine, which

also ran at 270 revolutions but was more heavily loaded. This load was also electrical and very steady, Fig. 138 being its diagram with the direct connection and 35 passages of the pencil.

Fig. 139 shows very prettily the effect of added clearance obtained by opening the cock *B*, leaving the passage to the indicator still direct.

Fig. 140 shows the diagram obtained with the indirect connection, the pencil passing 25 times over.

Fig. 141 shows all three diagrams on the same card.

Seven or eight feet of pipe is of course excessive for an indicator connection, though not much more so than 6 feet of steam hose. If such a difference as this exists with 8 feet there should be a visible difference with 4½ feet, or even with the ordinary side pipe on a long cylinder.

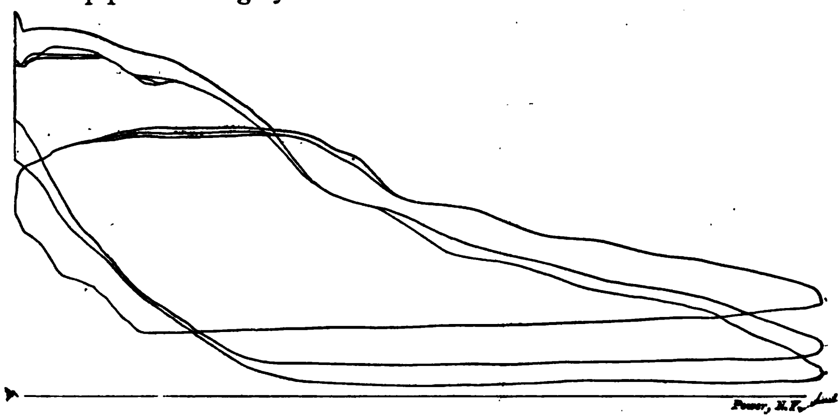


Fig. 141.

Fig. 134 is a photographic reproduction of the diagram obtained from the first engine with the direct connection, the pencil passing over it fully twenty times. A new card was placed upon the paper barrel and another diagram taken under the same conditions as Fig. 134. Then leaving the three-way cock so that the steam passed directly to the indicator, the cock *B* was opened, adding the pipe to the volume of the clearance, and another diagram was drawn upon the same card. The result is shown in Fig. 135, and is as would have been expected,—less realized pressure, lower compression, and higher terminal. For greater distinctness, we have dotted the line of the first diagram, which will be seen to be identical with Fig. 134.

CHAPTER XIX.

MEASURING THE CLEARANCE.

The clearance of a steam engine includes not only the space between the piston face and cylinder head, but all of the port or ports up to the valve face when the engine is on the dead center. It is necessary to know its exact amount whenever any accurate calculations are made concerning the action of the steam. It is usually expressed as a fraction of the volume displaced by one stroke of the piston, or what is equivalent to this, a per

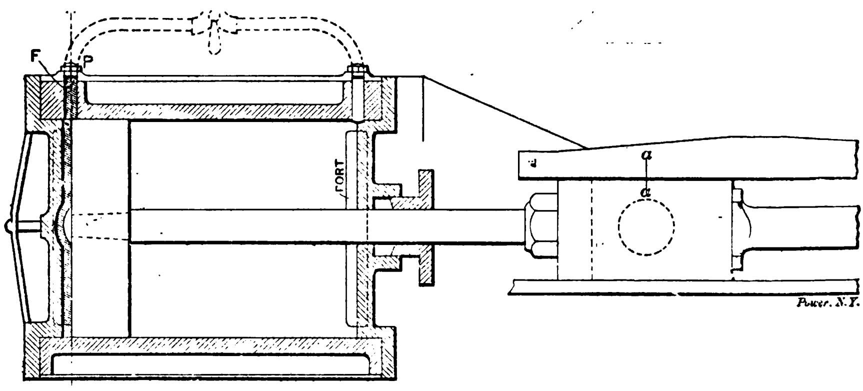


Fig. 142.

centage of the length of the stroke. Fig. 142 shows a single valve engine with the steam chest at the side of the cylinder, and the closely shaded portion represents the clearance.

If the valve and piston are tight, the amount of the clearance may be found both easily and accurately as follows:

Put the engine carefully on the dead center in the usual manner and set the valve so that it covers the port, blocking it, if necessary, to hold it up against the seat. Make a fine mark *a a* on the crosshead and guides.

Remove the indicator plug *P* and pour in enough water to fill the clearance space up to the under face *F* of the plug, which is the highest point of the clearance. Measure or weigh carefully, the amount of water poured in and make a note of it.

Now turn the engine over until the crosshead has moved 3 or 4 inches of its stroke and pour in a second quantity of water exactly equal to that required to fill the clearance space. Then back the engine up until the water rises again to the original level *F*. The crosshead and piston will now be in the position shown in Fig. 143 and the shaded portion will be filled with water. Make a second mark *b* on the guides opposite the mark *a* on the crosshead. The dotted line *XY*, Fig. 143, represents the original position of the crosshead and the space to the right of it

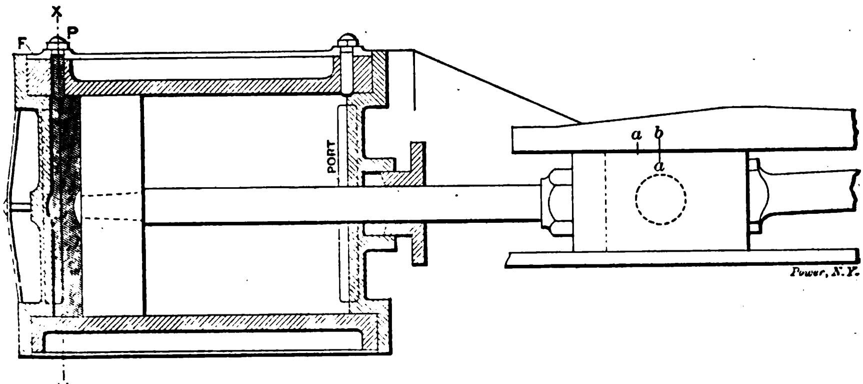


Fig. 143.

will be that occupied by the second quantity of water and will represent a volume equal to the clearance. The fraction of the stroke occupied by this equivalent volume will be the distance *a b* on the guides and all that we need to do to find the clearance in decimal parts of the stroke is to measure in inches the distance *a b* and divide it by the length of the stroke in inches.

For instance, if in an engine of 15 inches stroke we found the distance *a b* to be $1\frac{3}{16}$ inches (1.1875), the clearance would be

$\frac{1.1875}{15} = .0791$ or $7\frac{9}{10}$ per cent of the stroke.

In engines of the Corliss type, however, the indicator opening is not on top of the cylinder, but usually at the side, as shown in

Fig. 144. This objection can be overcome by screwing into the indicator elbow a short, vertical piece of pipe just long enough to bring the top end to the level of the valve face as in the figure. Then pour in the water until it overflows the top end of this pipe, leaving the steam valve open about as for lead to prevent entrapping air at the highest point. If this air were not allowed to escape, it would be compressed until its pressure equaled the slight head of water and we should not be able to fill the entire clearance space with water.

The distance *a b* on the guides is then found as before by pouring in a second quantity of water and bringing it to the original level. It is well to note here that if the second pouring is exactly equal to the first, we shall have put in too much by the quantity contained in the short piece of pipe from *P* to *T*,

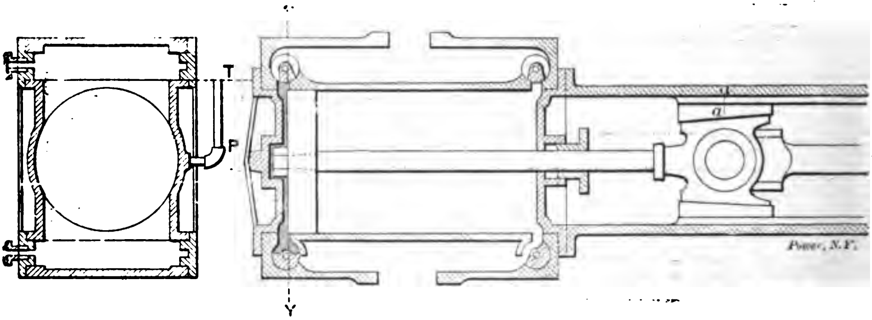


Fig. 144.

Fig. 144. This amount may be obtained by measurement before the pipe is screwed into place and should be deducted from the second pouring in order to correctly locate point *b*, Fig. 145. In the above method, it is not necessary to measure or weigh the quantity of water in any particular units; a mark on a bucket, any known number of canfuls or a balancing weight of unknown value will give us two equal quantities.

If a vessel graduated in U. S. liquid measure, i. e., quarts, pints and gills, be used to measure the first pouring, the second operation by which mark *b* was located, may be omitted and the clearance found by a simple calculation.

Suppose it required 3 quarts, 1 pint and 2 gills of water to fill the clearance of an engine 15 inches diameter by 15 inches

stroke. In U. S. liquid measure

4 gills $= 1$ pint
2 pints $= 1$ quart
4 quarts $= 1$ gallon

Since 1 gallon $= 231$ cubic inches,
1 gill $= 7.22$ cubic inches
1 pint $= 28.88$ cubic inches
1 quart $= 57.75$ cubic inches.

The volume of the clearance is then

$$\begin{array}{lrl} 3 \text{ quarts} & \times\ 57.75 = & 173.25 \\ 1 \text{ pint} & \times\ 28.88 = & 28.88 \\ 2 \text{ gills} & \times\ 7.22 = & 14.44 \\ \hline & \text{Total} = & 216.57 \text{ cu. in.} \end{array}$$

The cylinder area is $15^2 \times .7854 = 176.71$ square inches, and

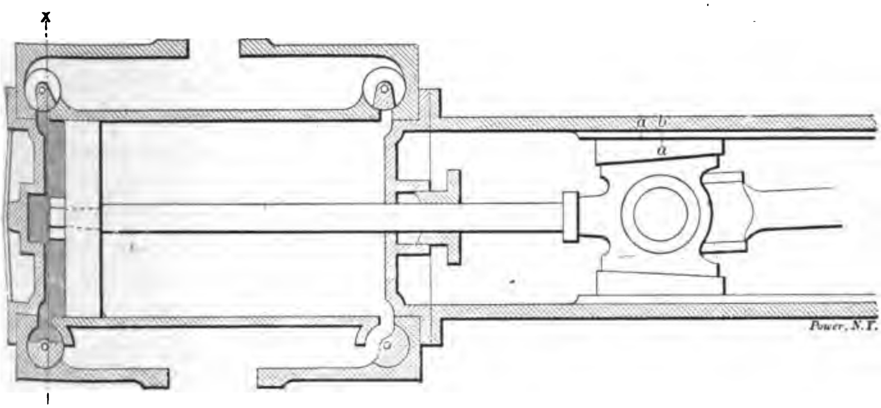


Fig. 145.

the piston displacement for one stroke is $176.71 \times 15 = 2,650.7$ cubic inches. Therefore the clearance is $216.6 \div 2,650.7 = .0817$ or $8\frac{17}{100}$ per cent of the stroke.

Even if the measuring apparatus is not graduated finer than pints, it is possible to estimate with reasonable accuracy to quarter pints, so that the error will not be serious.

There is another good way to find the clearance without locating point *b* on the guides; it requires only the use of a pair of avoirdupois scales, such as all grocers use, and a bucket holding

two or more times the water required to fill the clearance.

To illustrate more clearly, we will work out an example. Fill the bucket with water and weigh it carefully; let us assume that the bucket and water weigh 20 pounds. Now fill the clearance space from the bucket, taking care to spill none of the water, and again weight the bucket and the remaining water; suppose that it now weighs 12 pounds and 2 ounces. It has then required 20 pounds — 12 pounds 2 ounces = 7 pounds 14 ounces = $7\frac{14}{16}$ or 7.88 pounds of water to fill the clearance space. One cubic foot of water weighs at ordinary temperature 62½ pounds and contains 1,728 cubic inches; or one pound of water is then equivalent to $\frac{1,728}{62.5}=27.69$ cubic inches. The volume of the clearance is $7.88 \times 27.69 = 170.6$ cubic inches. The percentage of clearance is then found as before by dividing the clearance volume by the product of the piston area and stroke, i. e., by the piston displacement.

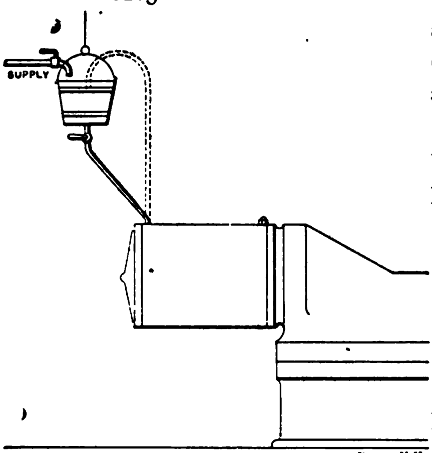


Fig. 146.

In engines having indicator openings on the side, a correction must be made for the short piece of pipe, as previously mentioned.

We now have three methods of finding clearance:

1.—By linear measure, using two equal quantities of water.
2.—By liquid measure.
3.—By weight.

There is still another method, which is as simple as any; it is shown in Fig. 146. A bucket or other vessel is suspended above the cylinder and a constant supply of water is furnished it by means of a hose or pipe. From the bottom or side of the bucket a small rubber hose or ⅛ inch pipe leads the water to the cylinder. The head of the water on the discharge end of the small pipe must be kept constant either by regulating the supply to

the bucket so as to keep the water level constant, or by allowing the bucket to overflow continually. If the latter is done, the overflowing water must not follow along the small pipe and so get into the cylinder. This can be prevented by using a siphon to supply the cylinder. The operation is as follows: Put the engine on the dead center and note the time in seconds required to fill the clearance space. Shut off the supply to the cylinder and put the engine on the other center. Then through the same pipe and under the same head fill the entire cylinder and clearance space up to the original level, noting separately the time in seconds required to fill the cylinder.

Since the quantity of water flowing through a constant opening under a constant head is exactly proportional to the time, the clearance is equal to the first period of flow divided by the second period.

For example, suppose it requires 1 minute and 25 seconds (85 seconds) to fill the clearance space and 28 minutes and 20 seconds (1,700 seconds) to fill the cylinder. The clearance is then $\frac{85}{1700} = .05$ or 5 per cent of the stroke.

The smaller the supply pipe to the cylinder, the longer it will take to fill the clearance space and the less the percentage of error in observation.

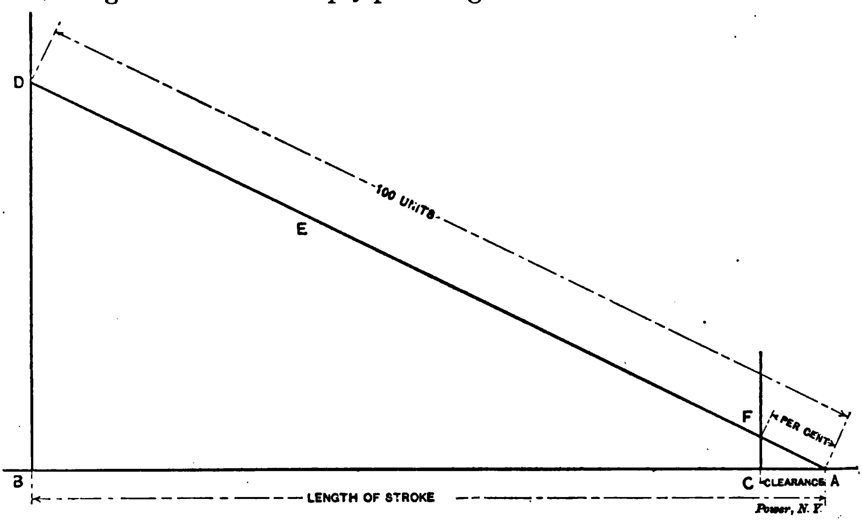
Various modifications of the details will suggest themselves for vertical engines, locomotive engines and others. In every case it is important to leave an opening for the escape of air at the highest point.

Suppose that instead of having plugs in the indicator openings the engine were provided with a ¾ inch standard indicator pipe and 3-way cock, as shown by the dotted lines in Fig. 142. The clearance space would then include that portion of the indicator pipe from the face of the 3-way cock to the cylinder connection. For a 15×15 inch engine, this additional amount would be about 11½ inches of ¾ inch standard pipe. The internal area of this pipe is .53 of a square inch, and the added clearance volume due to it is $.53 \times 11\frac{1}{2} = 6.10$ cubic inches. In finding the clearance of an engine equipped thus, the water should be poured in through the indicator connection until it is just visible from the top. When the side pipe is used and it is necessary to

use a riser the contents of the riser must be found separately and deducted.

The publication of the foregoing prepared by Mr. C. G. Robbins of the editorial department of POWER called out the following suggestion from Prof. John E. Sweet:

The engine valve and piston must be made tight and the engine set on the dead center as in any case. Set upon a platform or counter scale, a pail of water and an empty pail, and balance them by the weight on the scale, fill the clearance space from the pail of water, and then from an outside source put enough water in the empty pail to again balance the scale. Mark



GRAPHIC METHOD OF DETERMINING THE PER CENT OF CLEARANCE IN AN ENGINE.

Fig. 147.

the crosshead and guide, turn the engine forward a little way and put the water in the second pail in the cylinder, and turn the engine back until the water comes up in the indicator hole, and again mark the crosshead as was clearly explained in the foregoing.

In the case of a Corliss engine where a stand pipe is necessary to fill through the indicator hole, after the scale has been balanced with the pail of water, and empty pail as above described, take off the stand pipe, fill it with water and put it in the pail of water, then after putting on the stand pipe proceed as before.

So far we have in a simple way obtained two marks on the guide which truly represents the distance the piston has to travel to equal the clearance, and whether the result is in even inches, which would render it simple to determine the per cent, or in fractions, which would complicate the problem, the following graphic method answers equally well, and is readily performed by any one who can use a rule.

Draw a horizontal line as in Fig. 147, and lay off the stroke of the engine *A B* and draw the vertical line from *B*; at *C* draw another vertical line the same distance from *A* as the two lines marked on the guide. From *A* with 100 units of any convenient length measure up on the line *B*; that is to say, if the stroke of the engine be 11 inches measure up from *A* to some point on the line *F* to *D* 12½ inches, which is a hundred units of ⅛ inch each, then from *D* to *A* strike the straight line *E* and as many ⅛ inches as there are from *A* to *F*, so much will be the the per cent of clearance in the engine. If the stroke of the engine is between 13 and 18 inches, 18¾ inches may be used for the line *E* when $\frac{3}{16}$ of an inch will be the unit, or if from 18 to 24 inches, then 25 inches for the line *X* with ¼ inch as a unit and so on.

Of course this is not the mathematicians' way of doing things, but it eliminates many sources of error, is quick, easy to understand, and just as accurate as the man who does it is able to work, and that is the measure of accuracy in about everything.

This table of Physical Properties of steam was compiled by Wm. Kent, M. E. from various sources with a few changes, as follows: The figures for temperature, total heat, and latent heat are taken, up to 210 lbs. absolute pressure, from the tables in Porter's Steam-engine Indicator which tables have been widely accepted as standard by American engineers. The figures for total heat, given in the original as from 0° F., have been changed to heat above 32° F. The figures for weight per cubic foot and for cubic feet per pound have been taken from Dwelshauvers-Dery's table, Trans. A. S. M. E., Vol. XI., as being probably more accurate than those of Porter. The figures for relative volume are from Buel's table, in Dubois's translation of Weisbach, Vol. II. They agree quite closely with the relative volumes calculated from weights as given by Dery. From 211 to 219 lbs. the figures for temperature, total heat, and latent heat are from Dery's table; and from 220 to 1,000 lbs. all the figures are from Buel's table. The figures have not been carried out to as many decimal places as they are in most of the tables given by the different authorities; but any figure beyond the fourth significant figure is unnecessary in practice, and beyond the limit of error of the observations and the formulae from which the figures are derived.

 TABLE VII.—*Properties of Saturated Steam.*

Vacuum Gauge, Inches of Mercury.	Absolute Pressure, lbs. per square inch.	Temperature Fahrenheit.	Total Heat above 32° F. In the Water h Heat-units.	Total Heat above 32° F. In the Steam H Heat-units.	Latent Heat L. $= H - h$. Heat-units.	Relative Volume. Vol. of Water at 39° F. = 1.	Volume Cu. ft. in 1 lb. of Steam.	Weight of 1 cu. ft. Steam lb.
29.74	.089	32	0	1091.7	1091.7	208080	3333.3	.00030
29.67	.122	40	8	1094.1	1086.1	154330	2472.2	.00040
29.56	.176	50	18	1097.2	1079.2	107630	1724.1	.00058
29.40	.254	60	28.01	1100.2	1072.2	76370	1223.4	.00082
29.19	.359	70	38.02	1103.3	1065.3	54660	875.61	.00115
28.90	.502	80	48.04	1106.3	1058.3	39690	635.80	.00158
28.51	.692	90	58.06	1109.4	1051.3	29290	469.29	.00213
28.00	.943	100	68.08	1112.4	1044.4	21830	349.70	.00286
27.88	1	102.1	70.09	1113.1	1043.0	20623	334.23	.00299
25.85	2	126.3	94.44	1120.5	1026.0	10730	173.23	.00577
23.83	3	141.6	109.9	1125.1	1015.3	7325	117.93	.00848
21.78	4	153.1	121.4	1128.6	1007.2	5588	89.80	.01112
19.74	5	162.3	130.7	1131.4	1000.7	4530	72.50	.01373
17.70	6	170.1	138.6	1133.8	995.2	3816	61.10	.01631
15.67	7	176.9	145.4	1135.9	990.5	3302	53.00	.01887
13.63	8	182.9	151.5	1137.7	986.2	2912	46.60	.02149
11.60	9	188.3	156.9	1139.4	982.4	2607	41.82	.02391
9.56	10	193.2	161.9	1140.9	979.0	2361	37.80	.02641
7.52	11	197.8	166.5	1142.3	975.8	2159	34.61	.02889
5.49	12	202.0	170.7	1143.5	972.8	1990	31.90	.03136
3.45	13	205.9	174.7	1144.7	970.0	1846	29.58	.03381
1.41	14	209.6	178.4	1145.9	967.4	1721	27.50	.03625
Gauge Pressure lbs. per sq. in.	14.7	212	180.9	1146.6	965.7	1646	26.36	.03794
0.304	15	213.0	181.9	1146.9	965.0	1614	25.87	.03868
1.3	16	216.3	185.3	1147.9	962.7	1519	24.33	.04110
2.3	17	219.4	188.4	1148.9	960.5	1434	22.98	.04352
3.3	18	222.4	191.4	1149.8	958.3	1359	21.78	.04592
4.3	19	225.2	194.3	1150.6	956.3	1292	20.70	.04831
5.3	20	227.9	197.0	1151.5	954.4	1231	19.72	.05070
6.3	21	230.5	199.7	1152.2	952.6	1176	18.84	.05308
7.3	22	233.0	202.2	1153.0	951.8	1126	18.03	.05545
8.3	23	235.4	204.7	.7	949.1	1080	17.30	.05782
9.3	24	237.8	207.0	1154.5	947.4	1038	16.62	.06018
10.3	25	240.0	209.3	1155.1	945.8	998.4	15.99	.06253
11.3	26	242.2	211.5	.8	944.3	962.3	15.42	.06487
12.3	27	244.3	213.7	1156.4	942.8	928.8	14.88	.06721
13.3	28	246.3	215.7	1157.1	941.3	897.6	14.38	.06955
14.3	29	248.3	217.8	.7	939.9	868.5	13.91	.07188
15.3	30	250.2	219.7	1158.3	938.9	841.3	13.48	.07420

* See note on opposite page.

Gauge Pressure, lbs. per sq. in.	Absolute Pressure, lbs. per square inch.	Temperature Fahrenheit.	Total Heat above 32° F. In the Water h Heat-units.	Total Heat above 32° F. In the Steam H Heat units.	Latent Heat L, $= H - h$ Heat-units.	Relative Volume Vol. of Water at 39° F. = 1.	Volume Cu. ft. in 1 lb. of Steam.	Weight of 1 cu. ft. Steam lb.
16.3	31	252.1	221.6	.8	937.2	815.8	13.07	.07652
17.3	32	254.0	223.5	1159.4	935.9	791.8	12.68	.07884
18.3	33	255.7	225.3	.9	934.6	769.2	12.32	.08115
19.3	34	257.5	227.1	1160.5	933.4	748.0	11.98	.08346
20.3	35	259.2	228.8	1161.0	932.2	727.9	11.66	.08576
21.3	36	260.8	230.5	.5	931.0	708.8	11.36	.08806
22.3	37	262.5	232.1	1162.0	929.8	690.8	11.07	.09035
23.3	38	264.0	233.8	.5	928.7	673.7	10.79	.09264
24.3	39	265.6	235.4	.9	927.6	657.5	10.53	.09493
25.3	40	267.1	236.9	1163.4	926.5	642.0	10.28	.09721
26.3	41	268.6	238.5	.9	925.4	627.3	10.05	.09949
27.3	42	270.1	240.0	1164.3	924.4	613.3	9.83	.1018
28.3	43	271.5	241.4	.7	923.3	599.9	9.61	.1040
29.3	44	272.9	242.9	1165.2	922.3	587.0	9.41	.1063
30.3	45	274.3	244.3	.6	921.3	574.7	9.21	.1086
31.3	46	275.7	245.7	1166.0	920.4	563.0	9.02	.1108
32.3	47	277.0	247.0	.4	919.4	551.7	8.84	.1131
33.3	48	278.3	248.4	.8	918.5	540.9	8.67	.1153
34.3	49	279.6	249.7	1167.2	917.5	530.5	8.50	.1176
35.3	50	280.9	251.0	.6	916.6	520.5	8.34	.1198
36.3	51	282.1	252.2	1168.0	915.7	510.9	8.19	.1221
37.3	52	283.3	253.5	.4	914.9	501.7	8.04	.1243
38.3	53	284.5	254.7	.7	914.0	492.8	7.90	.1266
39.3	54	285.7	256.0	1169.1	913.1	484.2	7.76	.1288
40.3	55	286.9	257.2	.4	912.3	475.9	7.63	.1311
41.3	56	288.1	258.3	.8	911.5	467.9	7.50	.1333
42.3	57	289.1	259.5	1170.1	910.6	460.2	7.38	.1355
43.3	58	290.3	260.7	.5	909.8	452.7	7.26	.1377
44.3	59	291.4	261.8	.8	909.0	445.5	7.14	.1400
45.3	60	292.5	262.9	1171.2	908.2	438.5	7.03	.1422
46.3	61	293.6	264.0	.5	907.5	431.7	6.93	.1444
47.3	62	294.7	265.1	.8	906.7	425.2	6.82	.1466
48.3	63	295.7	266.2	1172.1	905.9	418.3	6.72	.1488
49.3	64	296.8	267.2	.4	905.2	412.3	6.62	.1511
50.3	65	297.8	268.3	.8	904.5	406.6	6.53	.1533
51.3	66	298.8	269.3	1173.1	903.7	400.8	6.43	.1555
52.3	67	299.8	270.4	.4	903.0	395.2	6.34	.1577
53.3	68	300.8	271.4	.7	902.3	389.8	6.25	.1599
54.3	69	301.8	272.4	1174.0	901.6	384.5	6.17	.1621
55.3	70	302.7	273.4	.3	900.9	379.3	6.09	.1643
56.3	71	303.7	274.4	.6	900.2	374.3	6.01	.1665
57.3	72	304.6	275.3	.8	899.5	369.4	5.93	.1687
58.3	73	305.6	276.3	1175.1	898.9	364.6	5.85	.1709
59.3	74	306.5	277.2	.4	898.2	360.0	5.78	.1731

Gauge Pressure, lbs. per sq. in.	Absolute Pressure, lbs. per square inch.	Temperature Fahrenheit.	Total Heat above 32° F. In the Water h Heat-units.	Total Heat above 32° F. In the Steam H Heat-units.	Latent Heat $L = H - h$. Heat-units.	Relative Volume. Vol. of Water at 32° F. = 1.	Volume. Cu. ft. in 1 lb. of Steam	Weight of 1 Cu. ft. Steam lb.
60.3	75	307.4	278.2	.7	897.5	355.5	5.71	.1753
61.3	76	308.3	279.1	1176.0	896.9	351.1	5.63	.1775
62.3	77	309.2	280.0	.2	896.2	346.8	5.57	.1797
63.3	78	310.1	280.9	.5	895.6	342.6	5.50	.1819
64.3	79	310.9	281.8	.8	895.0	338.5	5.43	.1840
65.3	80	311.8	282.7	1177.0	894.3	334.5	5.37	.1862
66.3	81	312.7	283.6	.3	893.7	330.6	5.31	.1884
67.3	82	313.5	284.5	.6	893.1	326.8	5.25	.1906
68.3	83	314.4	285.3	.8	892.5	323.1	5.18	.1928
69.3	84	315.2	286.2	1178.1	891.9	319.5	5.13	.1950
70.3	85	316.0	287.0	.3	891.3	315.9	5.07	.1971
71.3	86	316.8	287.9	.6	890.7	312.5	5.02	.1993
72.3	87	317.7	288.7	.8	890.1	309.1	4.96	.2015
73.3	88	318.5	289.5	1179.1	889.5	305.8	4.91	.2036
74.3	89	319.3	290.4	.3	888.9	302.5	4.86	.2058
75.3	90	320.0	291.2	.6	888.4	299.4	4.81	.2080
76.3	91	320.8	292.0	.8	887.8	296.3	4.76	.2102
77.3	92	321.6	292.8	1180.0	887.2	293.2	4.71	.2123
78.3	93	322.4	293.6	.3	886.7	290.2	4.66	.2145
79.3	94	323.1	294.4	.5	886.1	287.3	4.62	.2166
80.3	95	323.9	295.1	.7	885.6	284.5	4.57	.2188
81.3	96	324.6	295.9	1181.0	885.0	281.7	4.53	.2210
82.3	97	325.4	296.7	.2	884.5	279.0	4.48	.2231
83.3	98	326.1	297.4	.4	884.0	276.3	4.44	.2253
84.3	99	326.8	298.2	.6	883.4	273.7	4.40	.2274
85.3	100	327.6	298.9	.8	882.9	271.1	4.36	.2296
86.3	101	328.3	299.7	1182.1	882.4	268.5	4.32	.2317
87.3	102	329.0	300.4	.3	881.9	266.0	4.28	.2339
88.3	103	329.7	301.1	.5	881.4	263.6	4.24	.2360
89.3	104	330.4	301.9	.7	880.8	261.2	4.20	.2382
90.3	105	331.1	302.6	.9	880.3	258.9	4.16	.2403
91.3	106	331.8	303.3	1183.1	879.8	256.6	4.12	.2425
92.3	107	332.5	304.0	.4	879.3	254.3	4.09	.2446
93.3	108	333.2	304.7	.6	878.8	252.1	4.05	.2467
94.3	109	333.9	305.4	.8	878.3	249.9	4.02	.2489
95.3	110	334.5	306.1	1184.0	877.9	247.8	3.98	.2510
96.3	111	335.2	306.8	.2	877.4	245.7	3.95	.2531
97.3	112	335.9	307.5	.4	876.9	243.6	3.92	.2553
98.3	113	336.5	308.2	.6	876.4	241.6	3.88	.2574
99.3	114	337.2	308.8	.8	875.9	239.6	3.85	.2596
100.3	115	337.8	309.5	1185.0	875.5	237.6	3.82	.2617
101.3	116	338.5	310.2	.2	875.0	235.7	3.79	.2638
102.3	117	339.1	310.8	.4	874.5	233.8	3.76	.2660

Gauge Pressure. lbs. per sq. in.	Absolute Pressure, lbs. per square inch.	Temperature Fahrenheit.	Total Heat above 32° F. In the Water h Heat-units.	Total Heat above 32° F. In the Steam H Heat-units.	Latent Heat $L = H - h$. Heat-units.	Relative Volume. Vol. of water at 39° F. = 1.	Volume Cu. ft. in 1 lb. of Steam.	Weight of 1 cu. ft. Steam. lb.
103.3	118	339.7	311.5	.6	874.1	231.9	3.73	.2681
104.3	119	340.4	312.1	.8	873.6	230.1	3.70	.2703
105.3	120	341.0	312.8	.9	873.2	228.3	3.67	.2724
106.3	121	341.6	313.4	1186.1	872.7	226.5	3.64	.2745
107.3	122	342.2	314.1	.3	872.3	224.7	3.62	.2766
108.3	123	342.9	314.7	.5	871.8	223.0	3.59	.2788
109.3	124	343.5	315.3	.7	871.4	221.3	3.56	.2809
110.3	125	344.1	316.0	.9	870.9	219.6	3.53	.2830
111.3	126	344.7	316.6	1187.1	870.5	218.0	3.51	.2851
112.3	127	345.3	317.2	.3	870.0	216.4	3.48	.2872
113.3	128	345.9	317.8	.4	869.6	214.8	3.46	.2894
114.3	129	346.5	318.4	.6	869.2	213.2	3.43	.2915
115.3	130	347.1	319.1	.8	868.7	211.6	3.41	.2936
116.3	131	347.6	319.7	1188.0	868.3	210.1	3.38	.2957
117.3	132	348.2	320.3	.2	867.9	208.6	3.36	.2978
118.3	133	348.8	320.8	.3	867.5	207.1	3.33	.3000
119.3	134	349.4	321.5	.5	867.0	205.7	3.31	.3021
120.3	135	350.0	322.1	.7	866.6	204.2	3.29	.3042
121.3	136	350.5	322.6	.9	866.2	202.8	3.27	.3063
122.3	137	351.1	323.2	1189.0	865.8	201.4	3.24	.3084
123.3	138	351.8	323.8	.2	865.4	200.0	3.22	.3105
124.3	139	352.2	324.4	.4	865.0	198.7	3.20	.3126
125.3	140	352.8	325.0	.5	864.6	197.3	3.18	.3147
126.3	141	353.3	325.5	.7	864.2	196.0	3.16	.3169
127.3	142	353.9	326.1	.9	863.8	194.7	3.14	3190
128.3	143	354.4	326.7	1190.0	863.4	193.4	3.11	.3211
129.3	144	355.0	327.2	.2	863.0	192.2	3.09	.3232
130.3	145	355.5	327.8	.4	862.6	190.9	3.07	.3253
131.3	146	356.0	328.4	.5	862.2	189.7	3.05	.3274
132.3	147	356.6	328.9	.7	861.8	185.5	3.04	.3295
133.3	148	357.1	329.5	.9	861.4	187.3	3.02	.3316
134.3	149	357.6	330.0	1191.0	861.0	186.1	3.00	.3337
135.3	150	358.2	330.6	.2	860.6	184.9	2.98	.3358
136.3	151	358.7	331.1	.3	860.2	183.7	2.96	.3379
137.3	152	359.2	331.6	.5	859.9	182.6	2.94	.3400
138.3	153	359.7	332.2	.7	859.5	181.5	2.92	.3421
139.3	154	360.2	332.7	.8	859.1	180.4	2.91	.3442
140.3	155	360.7	333.2	1192.0	858.7	179.2	2.89	.3463
141.3	156	361.3	333.8	.1	858.4	178.1	2.87	.3483
142.3	157	361.8	334.3	.3	858.0	177.0	2.85	.3504
143.3	158	362.3	334.8	.4	857.6	175.0	2.84	.3525
144.3	159	362.8	335.3	.6	857.2	174.9	2.82	.3546
145.3	160	363.3	335.9	[illegible]	[illegible]	173.9	2.80	.3567

Gauge Pressure, lbs. per. sq. in.	Absolute Pressure, lbs. per. square inch.	Temperature Fahrenheit.	Total Heat above 32° F. In the Water h Heat-units.	Total Heat above 32° F. In the Steam H Heat-units.	Latent Heat $L = H - h$ Heat-units.	Relative Volume. Vol. of Water at 39° F. = 1.	Volume Cu. ft. in 1 lb. of Steam.	Weight of 1 cu. ft. Steam lb.
146.3	161	363.8	336.4	.9	856.5	172.9	2.79	.3588
147.3	162	364.3	336.9	1193.0	856.1	171.9	2.77	.3609
148.3	163	364.8	337.4	.2	855.8	171.0	2.76	.3630
149.3	164	365.3	337.9	.3	855.4	170.0	2.74	.3650
150.3	165	365.7	338.4	.5	855.1	169.0	2.72	.3671
151.3	166	366.2	338.9	.6	854.7	168.1	2.71	.3692
152.3	167	366.7	339.4	.8	854.4	167.1	2.69	.3713
153.3	168	367.2	339.9	.9	854.0	166.2	2.68	.3734
154.3	169	367.7	340.4	1194.1	853.6	165.3	2.66	.3754
155.3	170	368.2	340.9	.2	853.3	164.3	2.65	.3775
156.3	171	368.6	341.4	.4	852.9	163.4	2.63	.3796
157.3	172	369.1	341.9	.5	852.6	162.5	2.62	.3817
158.3	173	369.6	342.4	.7	852.3	161.6	2.61	.3838
159.3	174	370.0	342.9	.8	851.9	160.7	2.59	.3858
160.3	175	370.5	343.4	.9	851.6	159.8	2.58	.3879
161.3	176	371.0	343.9	1195.1	851.2	158.9	2.56	.3900
162.3	177	371.4	344.3	.2	850.9	158.1	2.55	.3921
163.3	178	371.9	344.8	.4	850.5	157.2	2.54	.3942
164.3	179	372.4	345.3	.5	850.2	156.4	2.52	.3962
165.3	180	372.8	345.8	.7	849.9	155.6	2.51	.3983
166.3	181	373.3	346.3	.8	849.5	154.8	2.50	.4004
167.3	182	373.7	346.7	.9	849.2	154.0	2.48	.4025
168.3	183	374.2	347.2	.1	848.9	153.2	2.47	.4046
169.3	184	374.6	347.7	1196.2	848.5	152.4	2.46	.4066
170.3	185	375.1	348.1	.3	848.2	151.6	2.45	.4087
171.3	186	375.5	348.6	.5	847.9	150.8	2.43	.4108
172.3	187	375.9	349.1	.6	847.6	150.0	2.42	.4129
173.3	188	376.4	349.5	.7	847.2	149.2	2.41	.4150
174.3	189	376.9	350.0	.9	846.9	148.5	2.40	.4170
175.3	190	377.3	350.4	1197.0	846.6	147.3	2.39	.4191
176.3	191	377.7	350.9	.1	846.3	147.0	2.37	.4212
177.3	192	378.2	351.3	.3	845.9	146.3	2.36	.4233
178.3	193	378.6	351.8	.4	845.6	145.6	2.35	.4254
179.3	194	379.0	352.2	.5	845.3	144.9	2.34	.4275
180.3	195	379.5	352.7	.7	845.0	144.2	2.33	.4296
181.3	196	380.0	353.1	.8	844.7	143.5	2.[illegible]2	.4317
182.3	197	380.3	353.6	.9	844.4	142.8	2.[illegible]1	.4337
183.3	198	380.7	354.0	1198.1	844.1	142.1	2.29	.4358
184.3	199	381.2	354.4	.2	843.7	141.4	2.28	.4379
185.3	200	381.6	354.9	.3	843.4	140.8	2.27	.4400
186.3	201	382.0	355.3	.4	843.1	140.1	2.26	.4420
187.3	202	382.4	355.8	.6	842.8	139.5	2.25	.4441
188.3	203	382.8	356.2	.7	842.5	138.8	2.24	.4462

Gauge Pressure, lbs. per sq. in.	Absolute Pressure, lbs. per square inch.	Temperature Fahrenheit.	Total Heat above 32° F. In the Water h Heat-units.	Total Heat above 32° F. In the Steam H Heat-units.	Latent Heat L. $= H - h$. Heat-units.	Relative Volume. Vol. of water at 39° F. = 1.	Volume Cu. ft. in 1 lb. of Steam.	Weight of 1 cu. ft. Steam, lb.
189.3	204	383.2	356.6	.8	842.2	138.1	2.23	.4482
190.3	205	383.7	357.1	1199.0	841.9	137.5	2.22	.4503
191.3	206	384.1	357.5	.1	841.6	136.9	2.21	.4523
192.3	207	384.5	357.9	.2	841.3	136.3	2.20	.4544
193.3	208	384.9	358.3	.3	841.0	135.7	2.19	.4564
194.3	209	385.3	358.8	.5	840.7	135.1	2.18	.4585
195.3	210	385.7	359.2	.6	840.4	134.5	2.17	.4605
196.3	211	386.1	359.6	.7	840.1	133.9	2.16	.4626
197.3	212	386.5	360.0	.8	839.8	133.3	2.15	.4646
198.3	213	386.9	360.4	.9	839.5	132.7	2.14	.4667
199.3	214	387.3	360.9	1200.1	839.2	132.1	2.13	.4687
200.3	215	387.7	361.3	.2	838.9	131.5	2.12	.4707
201.3	216	388.1	361.7	.3	838.6	130.9	2.12	.4728
202.3	217	388.5	362.1	.4	838.3	130.3	2.11	.4748
203.3	218	388.9	362.5	.6	838.1	129.7	2.10	.4768
204.3	219	389.3	362.9	.7	837.8	129.2	2.09	.4788
205.3	220	389.7	362.2*	1200.8	838.6*	128.7	2.06	.4852
215.3	230	393.6	366.2	1202.0	835.8	123.3	1.98	.5061
225.3	240	397.3	370.0	1203.1	833.1	118.5	1.90	.5270
235.3	250	400.9	373.8	1204.2	830.5	114.0	1.83	.5478
245.3	260	404.4	377.4	1205.3	827.9	109.8	1.76	.5686
255.3	270	407.8	380.9	1206.3	825.4	105.9	1.70	.5894
265.3	280	411.0	384.3	1207.3	823.0	102.3	1.64	.6101
275.3	290	414.2	387.7	1203.3	820.6	99.0	1.585	.6308
285.3	300	417.4	390.9	1209.2	818.3	95.8	1.535	.6515
335.3	350	43[illegible]	40[illegible].3	1213.7	807.5	82.7	1.325	.7545
385.3	400	44[illegible].9	41[illegible].8	1217.7	797.9	72.8	1.167	.8572
435.3	450	456.6	432.2	1221.3	789.1	65.1	1.042	.9595
485.3	500	467.4	443.5	1224.5	781.0	58.8	.942	1.062
535.3	550	477.5	451.1	1227.6	773.5	53.6	.859	1.164
585.3	600	486.9	464.2	1230.5	766.3	49.3	.790	1.266
635.3	650	495.7	473.6	1233.2	759.6	45.6	.731	1.368
685.3	700	504.1	482.4	1235.7	753.3	42.4	.680	1.470
735.3	750	512.1	490.9	1238.0	747.2	39.6	.636	1.572
785.3	800	519.6	498.9	1240.3	741.4	37.1	.597	1.674
835.3	850	526.8	506.7	1242.5	735.8	34.9	.563	1.776
885.3	900	533.7	514.0	1244.7	730.6	33.0	.532	1.878
935.3	950	540.3	521.3	1246.7	725.4	31.4	.505	1.980
985.3	1000	546.8	528.3	1248.7	720.3	30.0	.480	2.082

***The discrepancies at 205.3 lbs. gauge are due to the change from Dery's to Buel's figures.**

TABLE X.—Numbers, or Diameters of Circles. Circumferences, Areas, Squares, Cubes, Square Roots and Cube Roots.

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
1	3.1416	0.7854	1	1	1.000	1.000
2	6.28	3.14	4	8	1.414	1.259
3	9.42	7.07	9	27	1.732	1.442
4	12.57	12.57	16	64	2.000	1.587
5	15.71	19.63	25	125	2.236	1.709
6	18.85	28.27	36	216	2.449	1.817
7	21.99	38.48	49	343	2.645	1.912
8	25.13	50.27	64	512	2.828	2.000
9	28.27	63.62	81	729	3.000	2.080
10	31.42	78.54	100	1,000	3.162	2.154
11	34.56	95.03	121	1,331	3.316	2.223
12	37.70	113.10	144	1,728	3.464	2.289
13	40.84	132.73	169	2,197	3.605	2.351
14	43.98	153.94	196	2,744	3.741	2.410
15	47.12	176.71	225	3,375	3.872	2.466
16	50.26	201.06	256	4,096	4.000	2.519
17	53.41	226.98	289	4,913	4.123	2.571
18	56.55	254.47	324	5,832	4.242	2.620
19	59.69	283.53	361	6,859	4.358	2.668
20	62.83	314.16	400	8,000	4.472	2.714
21	65.97	346.36	441	9,261	4.582	2.758
22	69.11	380.13	484	10,648	4.690	2.802
23	72.26	415.48	529	12,167	4.795	2.843
24	75.40	452.39	576	13,824	4.898	2.884
25	78.54	490.87	625	15,625	5.000	2.924
26	81.68	530.93	676	17,576	5.099	2.962
27	84.82	572.56	729	19,683	5.196	3.000
28	87.96	615.75	784	21,952	5.291	3.036
29	91.11	660.52	841	24,389	5.385	3.072
30	94.25	706.86	900	27,000	5.477	3.107
31	97.39	754.77	961	29,791	5.567	3.141
32	100.53	804.25	1,024	32,768	5.656	3.174
33	103.67	855.30	1,089	35,937	5.744	3.207
34	106.81	907.92	1,156	39,304	5.830	3.239
35	109.96	962.11	1,225	42,875	5.916	3.271
36	113.10	1017.88	1,296	46,656	6.000	3.301
37	116.24	1075.21	1,369	50,653	6.082	3.332
38	119.38	1134.11	1,444	54,872	6.164	3.361
39	122.52	1194.59	1,521	59,319	6.244	3.391
40	125.66	1256.64	1,600	64,000	6.324	3.419
41	128.80	1320.25	1,681	68,921	6.403	3.448
42	131.95	1385.44	1,764	74,088	6.480	3.476
43	135.09	1452.20	1,849	79,507	6.557	3.508
44	138.23	1520.53	1,936	85,184	6.633	3.530
45	141.37	1590.43	2,025	91,125	6.708	3.556
46	144.51	1661.90	2,116	97,336	6.782	3.583
47	147.65	1734.94	2,209	103,823	6.855	3.608
48	150.80	1809.56	2,304	110,592	6.928	3.634
49	153.94	1885.74	2,401	117,649	7.000	3.659
50	157.08	1963.50	2,500	125,000	7.071	3.684

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
51	160.22	2042.82	2,601	132,651	7.141	3.708
52	163.36	2123.72	2,704	140,608	7.211	3.732
53	166.50	2206.18	2,809	148,877	7.280	3.756
54	169.65	2290.22	2,916	157,464	7.348	3.779
55	172.79	2375.83	3,025	166,375	7.416	3.802
56	175.93	2463.01	3,136	175,616	7.483	3.825
57	179.07	2551.76	3,249	185,193	7.549	3.848
58	182.21	2642.08	3,364	195,112	7.615	3.870
59	185.35	2733.97	3,481	205,379	7.681	3.892
60	188.50	2827.43	3,600	216,000	7.745	3.914
61	191.64	2922.47	3,721	226,981	7.810	3.936
62	194.78	3019.07	3,844	238,328	7.874	3.957
63	197.92	3117.25	3,969	250,047	7.937	3.979
64	201.06	3216.99	4,096	262,144	8.000	4.000
65	204.20	3318.31	4,225	274,625	8.062	4.020
66	207.34	3421.19	4,356	287,496	8.124	4.041
67	210.49	3525.65	4,489	300,763	8.185	4.061
68	213.63	3631.68	4,624	314,432	8.246	4.081
69	216.77	3739.28	4,761	328,509	8.306	4.101
70	219.91	3848.45	4,900	343,000	8.366	4.121
71	223.05	3959.19	5,041	357,911	8.426	4.140
72	226.19	4071.50	5,184	373,248	8.485	4.160
73	229.34	4185.39	5,329	389,017	8.544	4.179
74	232.48	4300.84	5,476	405,224	8.602	4.198
75	235.62	4417.86	5,625	421,875	8.660	4.217
76	238.76	4536.46	5,776	438,976	8.717	4.235
77	241.90	4656.63	5,929	456,533	8.744	4.254
78	245.04	4778.36	6,084	474,552	8.831	4.272
79	248.19	4901.67	6,241	493,039	8.888	4.290
80	251.33	5026.55	6,400	512,000	8.944	4.308
81	254.47	5153.00	6,561	531,441	9.000	4.326
82	257.61	5281.02	6,724	551,368	9.055	4.344
83	260.75	5410.61	6,889	571,787	9.110	4.362
84	263.89	5541.77	7,056	592,704	9.165	4.379
85	267.03	5674.50	7,225	614,125	9.219	4.396
86	270.18	5808.80	7,396	636,056	9.273	4.414
87	273.32	5944.68	7,569	658,503	9.327	4.431
88	276.46	6082.12	7,744	681,472	9.380	4.447
89	279.60	6221.14	7,921	704,969	9.433	4.461
90	282.74	6361.73	8,100	729,000	9.486	4.481
91	285.88	6503.88	8,281	753,571	9.539	4.497
92	289.03	6647.61	8,464	778,688	9.591	4.514
93	292.17	6792.91	8,649	804,357	9.643	4.530
94	295.31	6939.78	8,836	830,584	9.695	4.546
95	298.45	7088.22	9,025	857,375	9.746	4.562
96	301.59	7238.23	9,216	884,736	9.797	4.578
97	304.73	7389.81	9,409	912,673	9.848	4.594
98	307.88	7542.96	9,604	941,192	9.899	4.610
99	311.02	7697.69	9,801	970,299	9.949	4.626
100	314.16	7853.98	10,000	1,000,000	10.000	4.641
101	317.30	8011.85	10,201	1,030,301	10.049	4.657
102	320.41	8171.28	10,404	1,061,208	10.099	4.672
103	323.58	8332.29	10,609	1,092,727	10.148	4.687
104	326.73	8494.87	10,816	1,124,864	10.198	4.702
105	329.87	8659.01	11,025	1,157,625	10.246	4.717
106	333.01	8824.73	11,236	1,191,016	10.295	4.732

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
107	336.15	8992.02	11,449	1,225,043	10.344	4.747
108	339.29	9160.88	11,664	1,259,712	10.392	4.762
109	342.43	9331.32	11,881	1,295,029	10.440	4.776
110	345.57	9503.32	12,100	1,331,000	10.488	4.791
111	348.72	9676.89	12,321	1,367,631	10.535	4.805
112	351.86	9852.03	12,544	1,404,928	10.583	4.820
113	355.00	10028.75	12,769	1,442,897	10.630	4.834
114	358.14	10207.03	12,996	1,481,544	10.677	4.848
115	361.28	10386.89	13,225	1,520,875	10.723	4.862
116	364.42	10568.32	13,456	1,560,896	10.770	4.876
117	367.57	10751.32	13,689	1,601,613	10.816	4.890
118	370.71	10935.88	13,924	1,643,032	10.862	4.904
119	373.85	11122.02	14,161	1,685,159	10.908	4.918
120	376.99	11309.73	14,400	1,728,000	10.954	4.932
121	380.13	11499.01	14,641	1,771,561	11.000	4.946
122	383.27	11689.87	14,884	1,815,848	11.045	4.959
123	386.42	11882.29	15,129	1,860,867	11.090	4.973
124	389.56	12076.28	15,376	1,906,624	11.135	4.986
125	392.70	12271.85	15,629	1,953,125	11.180	5.000
126	395.84	12468.98	15,876	2,000,376	11.224	5.013
127	398.98	12667.69	16,129	2,048,383	11.269	5.026
128	402.12	12867.96	16,384	2,097,152	11.313	5.039
129	405.26	13069.81	16,641	2,146,689	11.357	5.052
130	408.41	13273.23	16,900	2,197,000	11.401	5.065
131	411.55	13478.22	17,161	2,248,091	11.445	5.078
132	414.69	13684.78	17,424	2,299,968	11.489	5.091
133	417.83	13892.91	17,689	2,352,637	11.532	5.104
134	420.97	14102.61	17,956	2,406,104	11.575	5.117
135	424.11	14313.88	18,225	2,460,375	11.618	5.129
136	427.26	14526.72	18,496	2,515,456	11.661	5.142
137	430.40	14741.14	18,769	2,571,353	11.704	5.155
138	433.54	14957.12	19,044	2,620,872	11.747	5.167
139	436.68	15174.68	19,321	2,685,619	11.789	5.180
140	439.82	15393.80	19,600	2,744,000	11.832	5.192
141	442.96	15614.50	19,881	2,803,221	11.874	5.204
142	446.11	15836.77	20,164	2,863,288	11.916	5.217
143	449.25	16060.61	20,449	2,924,207	11.958	5.229
144	452.39	16286.02	20,736	2,985,984	12.000	5.241
145	455.53	16513.00	21,025	3,048,625	12.041	5.253
146	458.67	16741.55	21,316	3,112,136	12.083	5.265
147	461.81	16971.67	21,609	3,176,523	12.124	5.277
148	464.96	17203.36	21,904	3,241,792	12.165	5.289
149	468.10	17436.62	22,201	3,307,949	12.206	5.301
150	471.24	17671.46	22,500	3,375,000	12.247	5.313
151	474.38	17907.86	22,801	3,442,951	12.288	5.325
152	477.52	18145.84	23,104	3,511,808	12.328	5.336
153	480.66	18385.39	23,409	3,581,577	12.369	5.348
154	483.80	18626.50	23,716	3,652,264	12.409	5.360
155	486.95	18869.19	24,025	3,723,875	12.449	5.371
156	490.09	19113.45	24,336	3,796,416	12.489	5.383
157	493.23	19359.28	24,649	3,869,893	12.529	5.394
158	496.37	19606.68	24,964	3,944,312	12.569	5.406
159	499.51	19855.65	25,281	4,019,679	12.609	5.417
160	502.65	20106.19	25,600	4,096,000	12.649	5.428
161	505.80	20358.34	25,921	4,173,281	12.688	5.440
162	508.94	20611.99	26,244	4,251,528	12.727	5.451

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
163	512.08	20867.24	26,569	4,330,747	12.767	5.462
164	515.22	21124.07	26,896	4,410,944	12.806	5.473
165	518.36	21382.46	27,225	4,495,125	12.845	5.484
166	521.50	21642.43	27,556	4,574,296	12.884	5.495
167	524.65	21903.97	27,889	4,657,463	12.922	5.506
168	527.79	22167.08	28,224	4,741,632	12.961	5.517
169	530.93	22431.76	28,561	4,826,809	13.000	5.528
170	534.07	22698.01	28,900	4,913,000	13.038	5.539
171	537.21	22965.83	29,241	5,000,211	13.076	5.550
172	540.35	23235.22	29,584	5,088,448	13.114	5.561
173	543.50	23506.18	29.929	5,177,717	13.152	5.572
174	546.64	23778.71	30,276	5,268,024	13.190	5.582
175	549.78	24052.82	30,625	5,359,375	13.228	5.593
176	552.92	24328.49	30,976	5,451,776	13.266	5.604
177	556.06	24605.79	31,329	5,545.233	13.304	5.614
178	559.20	24884,56	31,684	5,639,752	13.341	5.625
179	562.34	25164.94	32,041	5,735,339	13.379	5.635
180	565.49	25446 90	32,400	5,832,000	13.416	5.646
181	568.63	25730.43	32,761	5,929,741	13.453	5.656
182	571.77	26015.53	33,124	6,028,568	13.490	5.667
183	574.91	26302.20	33,489	6,128,487	13.527	5.677
184	578.05	26590.44	33,856	6,229,504	13.564	5.687
185	581.19	26880.25	34,225	6,331,625	13.601	5.698
186	584.34	27171.63	34,596	6,434,856	13.638	5.708
187	587.48	27464.59	34,969	6,539,203	13.674	5.718
188	590.62	27759.11	35,344	6,644,672	13.711	5.728
189	593.76	28055.21	35,721	6,751,269	13.747	5.738
190	596.90	28352.87	36,100	6,859,000	13.784	5.748
191	600.04	28652.11	36,481	6,967,871	13.820	5.758
192	603.19	28952.92	36.864	7,077,888	13.856	5.768
193	606.33	29255.30	37,249	7,189,057	13.892	5.778
194	609.47	29559.26	37,636	7,301,384	13.928	5.788
195	612.61	29864.77	38,025	7,414,875	13.964	5.798
196	615.75	30171.86	38,416	7,529,536	14.000	5.808
197	618.89	30480.52	38,809	7,645,373	14.035	5.818
198	622.03	30790.75	39,204	7,762,392	14.071	5.828
199	625.18	31102.55	39,601	7,880,599	14.106	5.838
200	628.32	31415.93	40,000	8,000,000	14.142	5.848
201	631.46	31730.87	40,401	8,120,601	14.177	5.857
202	634.60	32047.39	40,804	8,242,408	14.212	5.867
203	637.74	32365.47	41,209	8,365,427	14.247	5.877
204	640.88	32685.13	41,616	8,489,664	14.282	5.886
205	644.03	33006.36	42,025	8,615,125	14.317	5.896
206	647.17	33329.16	42,436	8,741,816	14.352	5.905
207	650.31	33653.53	42,849	8,869,743	14.387	5.915
208	653.45	33979.47	43.264	8,998,912	14.422	5.924
209	656.59	34806.98	43,681	9.123,329	14.456	5.934
210	659.73	34636.06	44,100	9,261,000	14.491	5.943
211	662.88	34966.71	44,521	9,393,931	14.525	5.953
212	666.02	35298.94	44,944	9,528,128	14.560	5.962
213	669.16	35632.73	45,369	9,663,597	14.594	5.972
214	672.30	35968.09	45,796	9,800,344	14.628	5.981
215	675.44	36305.03	46,225	9,938,375	14.662	5.990
216	678.58	36643.61	46,656	10,077,696	14.696	6.000
217	681.73	36983.61	47,089	10,218,313	14.730	6.009
218	684.87	37325.26	47,524	10,360,232	14.764	6.018

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
219	688.01	37668.48	47,961	10,503.459	14.798	6.027
220	691.15	..38013.27	.. 48,400	..10,648,000	14.832	6.036
221	694.29	38359.63	48,841	10,793,861	14.866	6.045
222	697.43	..38707.56	.. 49,284	..10.941,048	14.899	6.055
223	700.57	39057.07	49,729	11,089,567	14.933	6.064
224	703.72	..39408.14	.. 50,176	..11,239,424	14.966	6.073
225	706.86	39760.78	50,625	11,390,625	15.000	6.082
226	710.00	..40115.00	.. 51,076	..11,543,176	15.033	6.091
227	713.14	40470.78	51,529	11,697,083	15.066	6.100
228	716.28	..40828.14	.. 51,984	..11,852,352	15.099	6.109
229	719.42	41187.07	52,441	12,008,989	15.132	6.118
230	722.57	..41547.56	.. 52,900	..12,167,000	15.165	6.126
231	725.71	41909.63	53,361	12,326,391	15.198	6.135
232	728.85	..42273.27	.. 53,824	..12,487,168	15.231	6.144
233	731.99	42638.48	54,289	12,649,337	15.264	6.153
234	735.13	..43005.26	.. 54,756	..12,812,904	15.297	6.162
235	738.27	43373.61	55,225	12,977,875	15.329	6.171
236	741.42	..43743.54	.. 55,696	..13,144,256	15.362	6.179
237	744.56	44115.03	56,169	13,312,053	15.394	6.188
238	747.70	..44488.09	.. 56,644	..13,481,272	15 427	6.197
239	750.84	44862.73	57,121	13,651,919	15.459	6.205
240	753.98	..45238.93	.. 57,600	..13,824,000	15.491	6.214
241	757.12	45616.71	58,081	13,997,521	15.524	6.223
242	760.26	..45996.06	.. 58,564	..14,172,488	15.556	6.231
243	763.41	46376.98	59,049	14,348,907	15.588	6.240
244	766.55	..46759.47	.. 59,536	..14,526,784	15.620	6.248
245	769.69	47143.52	60,025	14,706,125	15.652	6.257
246	772.83	..47529.16	.. 60,516	..14,886,936	15.684	6.265
247	775.97	47916.36	61,009	15,069,223	15.716	6.274
248	779.11	..48305.13	.. 61,504	..15,252,992	15.748	6.282
249	782.26	48695.47	62,001	15,438,249	15.779	6.291
250	785.40	..49087.39	.. 62,500	..15,625,000	15.811	6.299
251	788.54	49480.87	63,001	15,813,251	15.842	6.307
252	791.68	..49875.92	.. 63,504	..16,003,008	15.874	6.316
253	794.82	50272.55	64,009	16,194,277	15.905	6.324
254	797.96	..50670.75	.. 64,516	..16,387,064	15.937	6.333
255	801.11	51070.52	65,025	16,581,375	15.968	6.341
256	804.25	..51471.86	.. 65,536	..16,777,216	16.000	6.349
257	807.39	51874.76	66,049	16,974,593	16.031	6.357
258	810.53	..52279.24	.. 66,564	..17.173,512	16.062	6.366
259	813.67	52685.29	67,081	17,373,979	16.093	6.374
260	816.81	..53092.96	.. 67,600	..17,576,000	16.124	6.382
261	819.96	53502.11	68,121	17,779,581	16.155	6.390
262	823.10	..53912.87	.. 68,644	..17,984,728	16.186	6.398
263	826.24	54325.21	69,169	18,191,447	16.217	6.406
264	829.38	..54739.11	.. 69,696	..18,399,744	16.248	6.415
265	832.52	55154.59	70,225	18,609,625	16.278	6.423
266	835.66	..55571.63	.. 70,756	..18,821,096	16.309	6.431
267	838.80	55990.25	71,289	19,034,163	16.340	6.439
268	841.95	..56410.44	.. 71,824	..19,248,832	16.370	6.447
269	845.09	56832.20	72,361	19,465,109	16.401	6.455
270	848.23	..57255.53	.. 72,900	..19,683,000	16.431	6.463
271	851.37	57680.43	73,441	19,902,511	16.462	6.471
272	854.51	..58106.90	.. 73,984	..20,123.648	16.492	6.479
273	857.65	58534.94	74,529	20,346,417	16.522	6.487
274	860.80	..58964 55	.. 75,076	..20,570.824	16.552	6.495

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
275	863.94	59395.74	75,625	20,796,875	16.583	6.502
276	867.08	59828.49	76,176	21,024,576	16.613	6.510
277	870.22	60262.82	76,729	21,253,933	16.643	6.518
278	873.36	60698.72	77,284	21,484,952	16.673	6.526
279	876.50	61136.18	77,841	21,717,639	16.703	6.534
280	879.65	61575.22	78,400	21,952,000	16.733	6.542
281	882.79	62015.82	78,961	22,188,041	16.763	6,549
282	885.93	62458.00	79,524	22,425,768	16.792	6.557
283	889.07	62901.75	80,089	22,665,187	16.822	6.565
284	892.21	63347.07	80,656	22,906,304	16.852	6.573
285	895.35	63793.97	81,225	23,149,125	16.881	6.580
286	898.49	64242.43	81,796	23,393,656	16.911	6.588
287	901.64	64692.46	82,369	23,639,903	16.941	6.596
288	904.78	65144.07	82,944	23,887,872	16.970	6.603
289	907.92	65597.24	83,521	24,137,569	17.000	6.611
290	911.06	66051.99	84,100	24,389,000	17.029	6.619
291	914.20	66508.30	84,681	24,642,171	17.059	6.627
292	917.34	66966.19	85,264	24,897,088	17.088	6.634
293	920.49	67425.65	85,849	25,153,757	17.117	6.642
294	923.63	67886.68	86,436	25,412,184	17.146	6.649
295	926.77	68349.28	87,025	25,672,375	17.176	6.657
296	929.91	68813.45	87,616	25,934,336	17.205	6.664
297	933.05	69279.19	88,209	26,198,073	17.234	6.672
298	936.19	69746.50	88,804	26,463,592	17.263	6.679
299	939.34	70215.38	89,401	26,730,899	17.292	6.687
300	942.48	70685.83	90,000	27,000,000	17.320	6.694
301	945.62	71157.86	90,601	27,270,901	17.349	6.702
302	948.76	71631.45	91,204	27,543,608	17.378	6.709
303	951.90	72106.62	91,809	27,818,127	17.407	6.717
304	955.04	72583.36	92,416	28,094,464	17.436	6.724
305	958.19	73061.66	93,025	28,372,625	17.464	6.731
306	961.33	73541.54	93,636	28,652,616	17.493	6.739
307	964.47	74022.99	94,249	28,934,443	17.521	6.746
308	967.61	74506.01	94,864	29,218,112	17.549	6.753
309	970.75	74990.60	95,481	29,503,629	17.578	6.761
310	973.89	75476.76	96,100	29,791,000	17.607	6.768
311	977.03	75964.50	96,721	30,080,231	17.635	6.775
312	980.18	76453.80	97,344	30,371,328	17.663	6.782
313	983.32	76944.67	97,969	30,664,297	17.692	6.789
314	986.46	77437.12	98,596	30,959,144	17.720	6.797
315	989.60	77931.13	99,225	31,255,875	17.748	6.804
316	992.74	78426.72	99,856	31,554,496	17.776	6.811
317	995.88	78923.88	100,489	31,855,013	17.804	6.818
318	999.03	79422.60	101,124	32,157,432	17.832	6.826
319	1002.17	79922.90	101,761	32,461,759	17.860	6.833
320	1005.31	80424.77	102,400	32,768,000	17.888	6.839
321	1008.45	80928.21	103,041	33,076,161	17.916	6.847
322	1011.59	81433.22	103,684	33,386,248	17.944	6.854
323	1014.73	81939.80	104,329	33,698,267	17.972	6.861
324	1017.88	82447.96	104,976	34,012,224	18.000	6.868
325	1021.02	82957.68	105,625	34,328,125	18.028	6.875
326	1024.16	83468.98	106,276	34,645,976	18.055	6.882
327	1027.30	83981.84	106,929	34,965,783	18.083	6.889
328	1030.44	84496.28	107,584	35,287,552	18.111	6.896
329	1033.58	85012.28	108,241	35,611,289	18.138	6.903
330	1036.73	85529.86	108,900	35,937,000	18.166	6.910

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
331	1039.87	86049.01	109,561	36,264,691	18.193	6.917
332	1043.01	..86569.73	110,224	.. 36,594,368	18.221	6.924
333	1046.15	87092.02	110,889	36,926,037	18.248	6.931
334	1049.29	..87615.88	111,556	.. 37,259,704	18.276	6.938
335	1052.43	88141.31	112,225	37,595,375	18.303	6.945
336	1055.57	..88668.31	112,896	.. 37,933,056	18.330	6.952
337	1058.72	89196.88	113,569	38,272,753	18.357	6.959
338	1061.86	..89727.03	114,244	.. 38,614,472	18.385	6.966
339	1065.00	90258.74	114,921	38,958,219	18.412	6.973
340	1068.14	..90792.03	115,600	.. 39,304,000	18.439	6.979
341	1071.28	91326.88	116,281	39,651,821	18.466	6.986
342	1074.42	..91863.31	116,964	.. 40,001,688	18.493	6.993
343	1077.57	92401.31	117,649	40,353,607	18.520	7.000
344	1080.71	..92940.88	118,336	.. 40,707,584	18.547	7.007
345	1083.85	93482.02	119,025	41,063,625	18.574	7.014
346	1086.99	..94024.73	119,716	.. 41,421,736	18.601	7.020
347	1090.13	94569.01	120,409	41,781,923	18.628	7.027
348	1093.27	..95114.86	121,104	.. 42,144,192	18.655	7.034
349	1096.42	95662.28	121,801	42,508,549	18.681	7.040
350	1099.56	..96211.28	122,500	.. 42,875,000	18.708	7.047
351	1102.70	96761.84	123,201	43,243,551	18.735	7.054
352	1105.84	..97314.76	123,904	.. 43,614,208	18.762	7.061
353	1108.98	97867.68	124,609	43,986,977	18.788	7.067
354	1112.12	..98422.96	125,316	.. 44,361,864	18.815	7.074
355	1115.26	98979.80	126,025	44,738,875	18.842	7.081
356	1118.41	..99538.22	126,736	.. 45,118,016	18.868	7.087
357	1121.55	100098.21	127,449	45,499,293	18.894	7.094
358	1124.69	100659.27	128,164	.. 45,882,712	18.921	7.101
359	1127.83	101222.90	128,881	46,268,279	18.947	7.107
360	1130.97	101787.60	129,600	.. 46,656,000	18.974	7.114
361	1134.11	102353.87	130,321	47,045,881	19.000	7.120
362	1137.26	102921.72	131,044	.. 47,437,928	19.026	7.127
363	1140.40	103491.13	131,769	47,832,147	19.052	7.133
364	1143.54	104062.12	132,496	.. 48,228,544	19.079	7.140
365	1146.68	104634.67	133,225	48,627,125	19.105	7.146
366	1149.82	105208.80	133,956	.. 49,027,896	19.131	7.153
367	1152.96	105784.49	134,689	49,430,863	19.157	7.159
368	1156.11	106361.76	135,424	.. 49,836,032	19.183	7.166
369	1159.25	106940.60	136,161	50,243,409	19.209	7.172
370	1162.39	107521.01	136,900	.. 50,653,000	19.235	7.179
371	1165.53	108102.99	137,641	51,064,811	19.261	7.185
372	1168.67	108686.54	138,384	.. 51,478,848	19.287	7.192
373	1171.81	109271.66	139,129	51,895,117	19.313	7.198
374	1174.96	109858.35	139,876	.. 52,313,624	19.339	7.205
375	1178.10	110446.62	140,625	52,734,375	19.365	7.211
376	1181.24	111036.45	141,376	.. 53,157,376	19.391	7.218
377	1184.38	111627.86	142,129	53,582,633	19.416	7.224
378	1187.52	112220.83	142,884	.. 54,010,152	19.442	7.230
379	1190.66	112815.38	143,641	54,439,939	19.468	7.237
380	1193.80	113411.49	144,400	.. 54,872,000	19.493	7.243
381	1196.95	114009.18	145,161	55,306,341	19.519	7.249
382	1200.09	114608.44	145,924	.. 55,742,968	19.545	7.256
383	1203.23	115209.27	146,689	56,181,887	19.570	7.262
384	1206.37	115811.67	147,456	.. 56,623,104	19.596	7.268
385	1209.51	116415.64	148,225	57,066,625	19.621	7.275
386	1212.65	117021.18	148,996	.. 57,512,456	19.647	7.281

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
387	1215.80	117628.30	149.769	57,960,603	19.672	7.287
388	1218.94	118236.98	150.544	58,411,072	19.698	7.294
389	1222.08	118847.24	151.321	58,863,869	19.723	7.299
390	1225.22	119459.06	152.100	59,319,000	19.748	7.306
391	1228.36	120072.46	152.881	59,776,471	19.774	7.312
392	1231.50	120687.42	153.664	60,236,288	19.799	7.319
393	1234.65	121303.96	154.449	60,698,457	19.824	7.325
394	1237.79	121922.07	155.236	61,162,984	19.849	7.331
395	1240.93	122541.75	156.025	61,629,875	19.875	7.337
396	1244.07	123163.00	156.816	62,099,136	19.899	7.343
397	1247.21	123785.82	157.609	62,570,773	19.925	7.349
398	1250.35	124410.21	158.404	63,044,792	19.949	7.356
399	1253.49	125036.17	159.201	63,521,199	19.975	7.362
400	1256.64	125663.71	160.000	64,000,000	20.000	7.368
401	1259.78	126292.81	160.801	64,481,201	20.025	7.374
402	1262.92	126923.48	161.604	64,964,808	20.049	7.380
403	1266.06	127553.73	162.409	65,450,827	20.075	7.386
404	1269.20	128189.55	163.216	65,939,264	20.099	7.392
405	1272.34	128824.93	164.025	66,430,125	20.125	7.399
406	1275.49	129461.89	164.836	66,923,416	20.149	7.405
407	1278.63	130100.42	165.649	67,419,143	20.174	7.411
408	1281.77	130740.52	166.464	67,911,312	20.199	7.417
409	1284.91	131382.19	167.281	68,417,929	20.224	7.422
410	1288.05	132025.43	168.100	68,921,000	20.248	7.429
411	1291.19	132670.24	168.921	69,426,531	20.273	7.434
412	1294.34	133316.63	169.744	69,934,528	20.298	7.441
413	1297.48	133964.58	170.569	70,444,997	20.322	7.447
414	1300.62	134614.10	171.396	70,957,944	20.347	7.453
415	1303.76	135265.20	172.225	71,473,375	20.371	7.459
416	1306.90	135917.86	173.056	71,991,296	20.396	7.465
417	1310.04	136572.10	173.889	72,511,713	20.421	7.471
418	1313.19	137227.91	174.724	73,034,632	20.445	7.477
419	1316.33	137885.29	175.561	73,560,059	20.469	7.483
420	1319.47	138544.24	176.400	74,088,000	20.494	7.489
421	1322.61	139204.70	177.241	74,618,461	20.518	7.495
422	1325.75	139866.85	178.084	75,151,448	20.543	7.501
423	1328.89	140530.51	178.929	75,686,967	20.567	7.507
424	1332.03	141195.74	179.776	76,225,024	20.591	7.513
425	1335.18	141862.54	180.625	76,765,625	20.615	7.518
426	1338.32	142530.92	181.476	77,308,776	20.639	7.524
427	1341.46	143200.86	182.329	77,854,483	20.664	7.530
428	1344.60	143872.38	183.184	78,402,752	20.688	7.536
429	1347.74	144545.46	184.041	78,953,589	20.712	7.542
430	1350.88	145220.12	184.900	79,507,000	20.736	7.548
431	1354.03	145896.35	185.761	80,062,991	20.760	7.554
432	1357.17	146574.15	186.624	80,621,568	20.785	7.559
433	1360.31	147253.52	187.489	81,182,737	20.809	7.565
434	1363.45	147934.46	188.356	81,746,504	20.833	7.571
435	1366.59	148616.97	189.225	82,312,875	20.857	7.577
436	1369.73	149301.05	190.096	82,881,856	20.881	7.583
437	1372.88	149986.70	190.969	83,453,453	20.904	7.588
438	1376.02	150673.93	191.844	84,027,672	20.928	7.594
439	1379.16	151362.72	192.721	84,604,519	20.952	7.600
440	1382.30	152053.08	193.600	85,184,000	20.976	7.606
441	1385.44	152745.02	194.481	85,766,121	21.000	7.612
442	1388.58	153438.53	195.364	86,350,388	21.024	7.617

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
443	1391.73	154133.60	196,249	86,938,307	21.047	7.623
444	1394.87	154830.25	197,136	87,528,384	21.071	7.629
445	1398.01	155528.47	198,025	88,121,125	21.095	7.635
446	1401.15	156228.26	198,916	88,716,536	21.119	7.640
447	1404.29	156929.62	199,809	89,314,623	21.142	7.646
448	1407.43	157632.55	200,704	89,915,392	21.166	7.652
449	1410.57	158337.06	201,601	90,518,849	21.189	7.657
450	1413.72	159043.13	202,500	91,125,000	21.213	7.663
451	1416.86	159750.77	203,401	91,733,851	21.237	7.669
452	1420.00	160459.99	204,304	92,345,408	21.260	7.674
453	1423.14	161170.77	205,209	92,959,677	21.284	7.680
454	1426.28	161883.13	206,106	93,576,664	21.307	7.686
455	1429.42	162597.06	207.025	94,196,375	21.331	7.691
456	1432.57	163312 55	207,936	94,818,816	21.354	7.697
457	1435.71	164029.62	208,849	95,443,993	21.377	7.703
458	1438.85	164748.26	209,764	96,071,912	21.401	7.708
459	1441.99	165468.47	210,681	96,702,579	21.424	7.714
460	1445.13	166190.25	211,600	97,336,000	21.447	7.719
461	1448.27	166913.60	212,521	97,972,181	21.471	7.725
462	1451.42	167638.53	213,444	98,611,128	21.494	7.731
463	1454.56	168365.02	214,369	99,252,847	21.517	7.736
464	1457.70	169093.08	215,296	99,897,345	21.541	7.742
465	1460.84	169822.72	216,225	100,544,625	21.564	7.747
466	1463.98	170553.92	217,156	101,194,696	21.587	7.753
467	1467.12	171286.70	218,089	101,847,563	21.610	7.758
468	1470.26	172021.05	219,024	102,503,232	21.633	7.764
469	1473.41	172756.97	219,961	103,161,709	21.656	7.769
470	1476.55	173494.45	220,900	103,823,000	21.679	7.775
471	1479.69	174233.51	221,841	104,487,111	21.702	7.780
472	1482.83	174974.14	222,784	105,154,048	21.725	7.786
473	1485 97	175716.35	223,729	105,823,817	21.749	7.791
474	1489.11	176460.12	224.676	106,496,424	21.771	7.797
475	1492.26	177205.46	225,625	107,171,875	21.794	7.802
476	1495.40	177952.37	226,576	107,850,176	21.817	7.808
477	1498.54	178700.86	227,529	108,531,333	21.840	7.813
478	1501.68	179450.91	228,484	109,215,352	21.863	7.819
479	1504 82	180202.54	229,441	109,902,239	21.886	7.824
480	1507.96	180955.74	230,400	110,592,000	21.909	7.830
481	1511.11	181710.50	231,361	111,284,641	21.932	7.835
482	1514.25	182466.84	232,324	111,980,168	21.954	7.840
483	1517.39	183224.75	233,289	112,678,587	21.977	7.846
484	1520.53	183984.23	234,256	113,379,904	22.000	7.851
485	1523.67	184745.28	235,225	114,084,125	22.023	7.857
486	1526.81	185507.90	236,196	114,791,256	22.045	7.862
487	1529.96	186272.10	237.169	115,501,303	22.069	7.868
488	1533.10	187037.86	238.144	116,214,272	22.091	7.873
489	1536.24	187805.19	239.121	116,936,169	22.113	7.878
490	1539.38	188574.10	240,100	117,649,000	22.136	7.884
491	1542.52	189344.57	241,081	118,370,771	22.158	7.889
492	1545.66	190116.62	242 064	119,095,488	22.181	7.894
493	1548.80	190890.24	243,049	119,823,157	22.204	7.899
494	1551.95	191665.43	244,036	120,553,784	22.226	7.905
495	1555.09	192442.19	245.025	121,287,375	22.248	7.910
496	1558.23	193220.51	246,016	122,023,936	22 271	7.915
497	1561.37	194000.42	247,009	122,763,473	22.293	7.921
498	1564.51	194781.89	248,004	123,505,992	22 316	7.926

Number, or Diameter	Circum-ference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
499	1567.65	195564.93	249,001	124,251,499	22.338	7.932
500	1570.80	196349.54	250,000	125,000,000	22.361	7.937
501	1573.94	197135.72	251,001	125,751,501	22.383	7.942
502	1577.08	197923.48	252,004	126,506,008	22.405	7.947
503	1580.22	198712.80	253,009	127,263,527	22.428	7.953
504	1583.36	199503.70	254,016	128,024,864	22.449	7.958
505	1586.50	200296.17	255,025	128,787,625	22.472	7.963
506	1589.65	201090.20	256,036	129,554,216	22.494	7.969
507	1592.79	201885.81	257,049	130,323,843	22.517	7.974
508	1595.93	202682.99	258,064	131,096,512	22.539	7.979
509	1599.07	203481.74	259,081	131,872,229	22.561	7.984
510	1602.21	204282.06	260,100	132,651,000	22.583	7.989
511	1605.35	205083.95	261,121	133,432,831	22.605	7.995
512	1608.49	205887.42	262,144	134,217,728	22.627	8.000
513	1611.64	206692.45	263,169	135,005,697	22.649	8.005
514	1614.78	207499.05	264,196	135,796,744	22.671	8.010
515	1617.92	208307.23	265,225	136,590,875	22.694	8.016
516	1621.06	209116.97	266,256	137,388,096	22.716	8.021
517	1624.20	209928.29	267,289	138,188,413	22.738	8.026
518	1627.34	210741.18	268,324	138,991,832	22.759	8.031
519	1630.49	211555.63	269,361	139,798,359	22.782	8.036
520	1633.63	212371.66	270,400	140,608,000	22.803	8.041
521	1636.77	213189.26	271,441	141,420,761	22.825	8.047
522	1639.91	214008.43	272,484	142,236,648	22.847	8.052
523	1643.05	214829.17	273,529	145,055,667	22.869	8.057
524	1646.19	215651.49	274,576	143,877,824	22.891	8.062
525	1649.34	216475.37	275,625	144,703,125	22.913	8.067
526	1652.48	217300.82	276,676	145,531,576	22.935	8.072
527	1655.62	218127.85	277,729	146,363,183	22.956	8.077
528	1658.76	218956.44	278,784	147,197,952	22.978	8.082
529	1661.90	219786.61	279,841	148,035,889	23.000	8.087
530	1665.04	220618.32	280,900	148,877,000	23.022	8.093
531	1668.19	221451.65	281,961	149,721,291	23.043	8.098
532	1671.33	222286.53	283,024	150,568,768	23.065	8.103
533	1674.47	223122.98	284,089	151,419,437	23.087	8.108
534	1677.61	223961.00	285,156	152,273,304	23.108	8.113
535	1680.75	224800.59	286,225	153,130,375	23.130	8.118
536	1683.89	225641.75	287,296	153,990,656	23.152	8.123
537	1687.04	226484.48	288,369	154,854,153	23.173	8.128
538	1690.18	227328.77	289,444	155,720,872	23.195	8.133
539	1693.32	228174.66	290,521	156,590,819	23.216	8.138
540	1696.46	229022.10	291,600	157,464,000	23.238	8.143
541	1699.60	229871.12	292,681	158,340,421	23.259	8.148
542	1702.74	230721.71	293,764	159,220,088	23.281	8.153
543	1705.88	231573.86	294,849	160,103,007	23.302	8.158
544	1709.03	232427.59	295,936	160,989,184	23.324	8.163
545	1712.17	233282.89	297,025	161,878,625	23.345	8.168
546	1715.31	234139.76	298,116	162,771,336	23.367	8.173
547	1718.45	234998.20	299,209	163,667,323	23.388	8.178
548	1721.59	235858.21	300,304	164,566,592	23.409	8.183
549	1724.73	236719.79	301,401	165,469,149	23.431	8.188
550	1727.88	237582.94	302,500	166.375,000	23.452	8.193
551	1731.02	238447.67	303,601	167,284,151	23.473	8.198
552	1734.16	239313.96	304,704	168,196,608	23.495	8.203
553	1737.30	240181.83	305.809	169,112,377	23.516	8.208
554	1740.44	241051.26	306,916	170,031,464	23.537	8.213

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
555	1743.58	241922.27	308,025	170,953,875	23.558	8.218
556	1746.73	242794.85	309,136	171,879,616	23.579	8.223
557	1749.87	243668.99	310,249	172,808,693	23.601	8.228
558	1753.00	244544.61	311,364	173,741,112	23.622	8.233
559	1756.15	245422.00	312,481	174,676,879	23.643	8.238
560	1759.29	246300.86	313,600	175,616,000	23.664	8.242
561	1762.43	247181.30	314,721	176,558,481	23.685	8.247
562	1765.57	248062.30	315,844	177,504,328	23.706	8.252
563	1768.72	248946.87	316,969	178,453,547	23.728	8.257
564	1771.86	249832.01	318,096	179,406,144	23.749	8.262
565	1775.00	250718.73	319,225	180,362,125	23.769	8.267
566	1778.14	251607.01	320,356	181,321,496	23.791	8.272
567	1781.28	252496.87	321,489	182,284,263	23.812	8.277
568	1784.42	253388.30	322,624	183,250,432	23.833	8.282
569	1787.57	254281.30	323,761	184,220,009	23.854	8.286
570	1790.71	255175.86	324,900	185,193,000	23.875	8.291
571	1793.85	256072.00	326,041	186,169,411	23.896	8.296
572	1796.99	256969.71	327,184	187,149,248	23.916	8.301
573	1800.13	257868.99	328,329	188,132,517	23.937	8.306
574	1803.27	258769.85	329,476	189,119,224	23.958	8.311
575	1806.42	259672.27	330,625	190,109,375	23.979	8.315
576	1809.56	260576.26	331,776	191,102,976	24.000	8.320
577	1812.70	261481.83	332,929	192,100,033	24.021	8.325
578	1815.84	262388.96	334,084	193,100,552	24.042	8.330
579	1818.98	263297.67	335,241	194,104,539	24.062	8.335
580	1822.12	264207.94	336,400	195,112,000	24.083	8.339
581	1825.26	265119.79	337,561	196,122,941	24.104	8.344
582	1828.41	266033.21	338,724	197,137,368	24.125	8.349
583	1831.55	266948.20	339,889	198,155,287	24.145	8.354
584	1834.69	267864.76	341,056	199,176,704	24.166	8.359
585	1837.83	268782.80	342,225	200,201,625	24.187	8.363
586	1840.97	269702.59	343,396	201,230,056	24.207	8.368
587	1844.11	270623.86	344,569	202,262,003	24.228	8.373
588	1847.26	271546.70	345,744	203,297,472	24.249	8.378
589	1850.40	272471.12	346,921	204,336,469	24.269	8.382
590	1853.54	273397.10	348,100	205,379,000	24.289	8.387
591	1856.68	274324.66	349,281	206,425,071	24.310	8.392
592	1859.82	275253.78	350,464	207,474,688	24.331	8.397
593	1862.96	276184.48	351,649	208,527,857	24.351	8.401
594	1866.11	277116.75	352,836	209,584,584	24.372	8.406
595	1869.25	278050.59	354,025	210,644,875	24.393	8.411
596	1872.39	278985.99	355,216	211,708,736	24.413	8.415
597	1875.53	279922.97	356,409	212,776,173	24.433	8.420
598	1878.67	280861.53	357,604	213,847,192	24.454	8.425
599	1881.81	281801.65	358,801	214,921,799	24.474	8.429
600	1884.96	282743.34	360,000	216,000,000	24.495	8.434
601	1888.10	283686.60	361,201	217,081,801	24.515	8.439
602	1891.24	284631.44	362,404	218,167,208	24.536	8.444
603	1894.38	285577.84	363,609	219,256,227	24.556	8.448
604	1897.52	286525.82	364,816	220,348,864	24.576	8.453
605	1900.66	287475.36	366,025	221,445,125	24.597	8.458
606	1903.80	288426.48	367,236	222,545,016	24.617	8.462
607	1906.95	289379.17	368,449	223,648,543	24.637	8.467
608	1910.09	290333.43	369,664	224,755,712	24.658	8.472
609	1913.23	291289.26	370,881	225,866,529	24.678	8.476
610	1916.37	292246.66	372,100	226,981,000	24.698	8.481

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
611	1919.51	293205.63	373,321	228,099,131	24.718	8.485
612	1922.65	294166.17	374,544	229,220,928	24.739	8.490
613	1925.80	295128.28	375,769	230,346,397	24.758	8.495
614	1928.94	296091.97	376,996	231,475,544	24.779	8.499
615	1932.08	297057.22	378,225	232,608,375	24.799	8.504
616	1935.22	298024.05	379,456	233,744,896	24.819	8.509
617	1938.36	298992.44	380,689	234,885,113	24.839	8.513
618	1941.50	299962.41	381,924	236,029,032	24.859	8.518
619	1944.65	300933.95	383,161	237,176,659	24.879	8.522
620	1947.79	301907.05	384,400	238,628,000	24.899	8.527
621	1950.93	302881.73	385,641	239,483,061	24.919	8.532
622	1954.07	303857.98	386,884	240,641,848	24.939	8.536
623	1957.21	304835.80	388,129	241,804,367	24.959	8.541
624	1960.35	305815.20	389,376	242,970,624	24.980	8.545
625	1963.50	306796.16	390,625	244,140,625	25.000	8.549
626	1966.64	307778.69	391,876	245,314,376	25.019	8.554
627	1969.78	308762.79	393,129	246,491,883	25.040	8.559
628	1972.92	309748.47	394,384	247,673,152	25.059	8.563
629	1976.06	310735.71	395,641	248,858,189	25.079	8.568
630	1979.20	311724.53	396,900	250,047,000	25.099	8.573
631	1982.34	312714.92	398,161	251,239,591	25.119	8.577
632	1985.49	313706.88	399,424	252,435,968	25.139	8.582
633	1988.63	314700.40	400,689	253,636,137	25.159	8.586
634	1991.77	315695.50	401,956	254,840,104	25.179	8.591
635	1994.91	316692.17	403,225	256,047,875	25.199	8.595
636	1998.05	317690.42	404,496	257,259,456	25.219	8.599
637	2001.19	318690.23	405,769	258,474,853	25.239	8.604
638	2004.34	319691.61	407,044	259,694,072	25.259	8.609
639	2007.48	320694.56	408,321	260,917,119	25.278	8.613
640	2010.62	321699.09	409,600	262'144,000	25.298	8.618
641	2013.76	322705.18	410,881	263,374,721	25.318	8.622
642	2016.90	323712.85	412,164	264,609,288	25.338	8.627
643	2020.04	324722.09	413,449	265,847,707	25.357	8.631
644	2023.19	325732.89	414,736	267,089,984	25.377	8.636
645	2026.33	326745.27	416,025	268,336,125	25.397	8.640
646	2029.47	327759.22	417,316	269,586,136	25.416	8.644
647	2032.61	328774.74	418,609	270,840,023	25.436	8.649
648	2035.75	329791.83	419,904	272,097,792	25.456	8.653
649	2038.89	330810.49	421,201	273,359,449	25.475	8.658
650	2042.04	331830.72	422,500	274,625,000	25.495	8.662
651	2045.18	332852.53	423,801	275,894,451	25.515	8.667
652	2048.32	333875.90	425,104	277,167,808	25.534	8.671
653	2051.46	334900.85	426,409	278,445,077	25.554	8.676
654	2054.60	335927.36	427,716	279,726,264	25.573	8.680
655	2057.74	386955.45	429,025	281,011,375	25.593	8.684
656	2060.88	337985.10	430,336	282,300,416	25.612	8.689
657	2064.03	339016.33	431,649	283,593,393	25.632	8.693
658	2067.17	340049.13	432,964	284,890,312	25.651	8.698
659	2070.31	341083.50	434,281	286,191,179	25.671	8.702
660	2073.45	342119.44	435,600	287,496,000	25.690	8.706
661	2076.59	343156.95	433,921	288,804,781	25.710	8.711
662	2079.73	344196.03	438,244	290,117,528	25.730	8.715
663	2082.88	345236.69	439,569	291,434,247	25.749	8.719
664	2086.02	346278.91	440,896	292,754,944	25.768	8.724
665	2089.16	347322.70	442,225	294,079,625	25.787	8.728
666	2092.30	348368.07	443,556	295,408,296	25.807	8.733

Number or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
667	2095.44	349415.00	444,889	296,740,963	25.826	8.737
668	2098.58	350463.51	446,224	298,077,632	25.846	8.742
669	2101.73	351513.59	447,561	299,418,309	25.865	8.746
670	2104.87	352565.24	448,900	300,763,000	25.884	8.750
671	2108.01	353618.43	450,241	302,111,711	25.904	8.753
672	2111.15	354673.24	451,584	303,464,448	25.923	8.759
673	2114.29	355729.60	452,929	304,821,217	25.942	8.763
674	2117.43	356787.54	454,276	306,182,024	25.961	8.768
675	2120.58	357847.04	455,625	307,546,875	25.981	8.772
676	2123.72	358908.11	456,976	308,915,776	26.000	8.776
677	2126.86	359970.75	458,329	310,288,733	26.019	8.781
678	2130.00	361034.97	459,684	311,665,752	26.038	8.785
679	2133.14	362100.75	461,041	313,046,839	26.058	8.789
680	2136.28	363168.11	462,400	314,432,000	26.077	8.794
681	2139.42	364237.04	463,761	315,821,241	26.096	8.798
682	2142.57	365307.54	465,124	317,214,568	26.115	8.802
683	2145.71	366379.60	466,489	318,611,987	26.134	8.807
684	2148.85	367453.24	467,856	320,013,504	26.153	8.811
685	2151.99	368528.45	469,225	321,419,125	26.172	8.815
686	2155.13	369605.23	470,596	322,828,856	26.192	8.819
687	2158.27	370683.59	471,969	324,242,703	26.211	8.824
688	2161.42	371763.51	473,344	325,660,672	26.229	8.828
689	2164.56	372845.00	474,721	327,082,769	26.249	8.832
690	2167.70	373928.07	476,100	328,509,000	26.268	8.836
691	2170.84	375012.70	477,481	329,939,371	26.287	8.841
692	2173.98	376098.91	478,864	331,373,888	26.306	8.845
693	2177.12	377186.68	480,249	332,812,557	26.325	8.849
694	2180.27	378276.03	481,636	334,255,384	26.344	8.853
695	2183.41	379366.95	483,025	335,702,375	26.363	8.858
696	2186.55	380459.44	484,416	337,153,536	26.382	8.862
697	2189.69	381553.50	485,809	338,608,873	26.401	8.866
698	2192.83	382649.43	487,204	340,068,392	26.419	8.870
699	2195.97	383746.33	488,601	341,532,099	26.439	8.875
700	2199.12	384845.10	490,000	343,000,000	26.457	8.879
701	2202.26	385945.44	491,401	344,472,101	26.476	8.883
702	2205.40	387047.36	492,804	345,948,088	26.495	8.887
703	2208.54	388150.84	494,209	347,428,927	26.514	8.892
704	2211.68	389255.90	495,616	348,913,664	26.533	8.896
705	2214.82	390362.52	497,025	350,402,625	26.552	8.900
706	2217.96	391470.32	498,436	351,895,816	26.571	8.904
707	2221.11	392580.49	499,849	353,393,243	26.589	8.908
708	2224.25	393691.83	501,264	354,894,912	26.608	8.913
709	2227.39	394804.74	502,681	356,400,829	26.627	8.917
710	2230.53	395919.21	504,100	357,911,000	26.644	8.921
711	2233.67	397035.27	505,521	359,425,431	26.664	8.925
712	2236.81	398152.89	506,944	360,944,128	26.683	8.929
713	2239.96	399272.08	508,369	362,467,097	26.702	8.934
714	2243.10	400392.84	509,796	363,994,344	26.721	8.938
715	2246.24	401515.18	511,225	365,525,875	26.739	8.942
716	2249.38	402639.08	512,656	367,061,696	26.758	8.946
717	2252.52	403764.56	514,089	368,601,813	26.777	8.950
718	2255.66	404891.60	515,524	370,146,232	26.795	8.954
719	2258.81	406020.22	516,961	371,694,959	26.814	8.959
720	2261.95	407150.41	518,400	373,248,000	26.833	8.963
721	2265.09	408282.17	519,841	374,805,361	26.851	8.967
722	2268.23	409415.50	521,284	376,367,048	26.870	8.971

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
723	2271.37	410550.40	522,729	377,933,067	26.889	8.975
724	2274.51	411686.87	524,126	379,503,424	26.907	8.979
725	2277.66	412824.91	525,625	381,078,125	26.926	8.983
726	2280.80	413964.52	527,076	382,657.176	26.944	8.988
727	2283.94	415105.71	528,529	384,240,583	26.963	8.992
728	2287.08	416248.46	529,984	385,828,352	26.991	8.996
729	2290.22	417392.79	531,441	387,420,489	27.000	9.000
730	2293.36	418538.68	532,900	389,017,000	27.018	9.004
731	2296.50	419686.15	534,361	390,617,891	27.037	9.008
732	2299.65	420835.19	535,824	392,223,168	27.055	9.012
733	2302.79	421985.79	537,289	393,832,837	27.074	9.016
734	2305.93	423137.97	538,756	395,446,904	27.092	9.020
735	2309.07	424291.72	540,225	397,065,375	27.111	9.025
736	2312.21	425447.04	541,696	398,688,256	27.129	9.029
737	2315.35	426603.93	543,169	400,315,553	27.148	9.033
738	2318.50	427762.40	544,644	401.947,272	27.166	9.037
739	2321.64	428922.43	546,121	403,583,419	27.184	9.041
740	2324.78	430084.03	547,600	405,224,000	27.203	9.045
741	2327.92	431247.21	549,081	406,869,021	27.221	9.049
742	2331.06	432411.95	550,564	408,518,488	27.239	9.053
743	2334.20	433578.27	552,049	410,172,407	27.258	9.057
744	2337.35	434746.16	553,536	411,830,784	27.276	9.061
745	2340.49	435915.62	555,025	413,493,625	27.295	9.065
746	2343.63	437086.64	556,516	415,160,936	27.313	9.069
747	2346.77	438259.24	558,009	416,832,723	27.331	9.073
748	2349.91	439433.41	559,504	418,508,992	27.349	9.077
749	2353.05	440609.16	561,001	420,189,749	27.368	9.081
750	2356.20	441786.47	562,500	421.875,000	27.386	9.086
751	2359.34	442965.35	564,001	423,564,751	27.404	9.089
752	2362.48	444145.80	565,504	424.525,900	27.423	9.094
753	2365.62	445327.83	567,009	426,957,777	27.441	9.098
754	2368.76	446511.42	568,516	428,661,064	27.459	9.102
755	2371.90	447696.59	570,025	430,368,875	27.477	9.106
756	2375.04	448883.32	571,536	432,081,216	27.495	9.109
757	2378.19	450071.63	573,049	433.798,093	27.514	9.114
758	2381.33	451261.51	574,564	435,519,512	27.532	9.118
759	2384.47	452452.96	576,081	437,245,479	27.549	9.122
760	2387.61	453645.98	577,600	438,976,000	27.568	9.126
761	2390.75	454840.57	579,121	440.711,081	27.586	9.129
762	2393.89	456036.73	580,644	442,450,728	27.604	9.134
763	2397.04	457234.46	582,169	444,194,947	27.622	9.138
764	2400.18	458433.77	583,696	445,943,744	27.640	9.142
765	2403.32	459634.64	585,225	447,697,125	27.659	9.146
766	2406.46	460837.08	586,756	449,455,096	27.677	9.149
767	2409.60	462041.10	588,289	451,217,663	27.695	9.154
768	2412.74	463246.69	589,824	452,984,832	27.713	9.158
769	2415.89	464453.84	591,361	454,756,609	27.731	9.162
770	2419.03	465662.57	592,900	456,533,000	27.749	9.166
771	2422.17	466872.87	594,441	458,314,011	27.767	9.169
772	2425.31	468084.74	595,984	460,099,648	27.785	9.173
773	2428.45	469298.18	597,529	461,889,917	27.803	9.177
774	2431.59	470513.19	599,076	463,684,824	27.821	9.181
775	2434.73	471729.77	600,625	465.484,375	27.839	9.185
776	2437.88	472947.92	602,176	467,288,576	27.857	9.189
777	2441.02	474167.65	603.729	469,097,433	27.875	9.193
778	2444.16	475388.94	605.284	470.910.952	27.893	9.197

Number or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
779	2447.30	476611.81	606,841	472,729,139	27.910	9.201
780	2450.44	477836.24	608,400	474,552,000	27.928	9.205
781	2453.58	479062.25	609,961	476,379,541	27.946	9.209
782	2456.73	480289.83	611,524	478,211,768	27.964	9.213
783	2459.87	481518.97	613,089	480,048,687	27.982	9.217
784	2463.01	482749.69	614,656	481,890,304	28.000	9.221
785	2466.15	483981.98	616,225	483,736,025	28.017	9.225
786	2469.29	485215.84	617,796	485,587,656	28.036	9.229
787	2472.43	486451.28	619,369	487,443,403	28.053	9.233
788	2475.58	487688.28	620,944	489,303,872	28.071	9.237
789	2478.72	488926.85	622,521	491,169,069	28.089	9.240
790	2481.86	490166.99	624,100	493,039,000	28.107	9.244
791	2485.00	491408.71	625,681	494,913,671	28.125	9.248
792	2488.14	492651.99	627,264	496,793,088	28.142	9.252
793	2491.28	493896.85	628,849	498,677,257	28.160	9.256
794	2494.43	495143.28	630,436	500,566,184	28.178	9.260
795	2497.57	496391.27	632,025	502,459,875	28.196	9.264
796	2500.71	497640.84	633,616	504,358,336	28.213	9.268
797	2503.85	498891.98	635,209	506,261,573	28.231	9.271
798	2506.99	500144.69	636,804	508,169,592	28.249	9.275
799	2510.13	501398.97	638,401	510,082,399	28.266	9.279
800	2513.27	502654.82	640,000	512,000,000	28.284	9.283
801	2516.42	503912.25	641,601	513,922,401	28.302	9.287
802	2519.56	505171.24	643,204	515,849,608	28.319	9.291
803	2522.70	506431.80	644,809	517,781,627	28.337	9.295
804	2525.84	507693.94	646,416	519,718,464	28.355	9.299
805	2528.98	508957.65	648,025	521,660,125	28.372	9.302
806	2532.12	510222.92	649,636	523,606,616	28.390	9.306
807	2535.27	511489.77	651,249	525,557,943	28.408	9.310
808	2538.41	512758.19	652,864	527,514,112	28.425	9.314
809	2541.55	514028.19	654,481	529,474,129	28.443	9.318
810	2544.09	515299.74	656,100	531,441,000	28.460	9.321
811	2547.83	516572.86	657,721	533,411,731	28.478	9.325
812	2550.97	517847.57	659,344	535,387,328	28.496	9.329
813	2554.12	519123.84	660,969	537,366,797	28.513	9.333
814	2557.26	520401.68	662,596	539,353,144	28.531	9.337
815	2560.40	521681.10	664,225	541,343,375	28.548	9.341
816	2563.54	522962.08	665,856	543,338,496	28.566	9.345
817	2566.68	524244.63	667,489	545,338,513	28.583	9.348
818	2569.82	525528.76	669,124	547,343,432	28.601	9.352
819	2572.96	526814.46	670,761	549,353,259	28.618	9.356
820	2576.11	528101.73	672,400	551,368,000	28.636	9.360
821	2579.25	529390.56	674,041	553,387,661	28.653	9.364
822	2582.39	530680.97	675,684	555,412,248	28.670	9.367
823	2585.53	531972.95	677,329	557,441,767	28.688	9.371
824	2588.67	533266.50	678,976	559,476,224	28.705	9.375
825	2591.81	534561.63	680,625	561,515,625	28.723	9.379
826	2594.96	535858.32	682,276	563,559,976	28.740	9.383
827	2598.10	537156.58	683,929	565,609,283	28.758	9.386
828	2601.24	538456.41	685,584	567,663,552	28.775	9.390
829	2604.38	539757.82	687,241	569,722,789	28.792	9.394
830	2607.52	541060.79	688,900	571,787,000	28.810	9.398
831	2610.66	542365.34	690,561	573,856,191	28.827	9.401
832	2613.81	543671.46	692,224	575,930,368	28.844	9.405
833	2616.95	544979.15	693,889	578,009,537	28.862	9.409
834	2620.09	546288.40	695,556	580,093,704	28.879	9.413

Number or Diameter.	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
835	2623.23	547599.23	697,225	582,182,875	28.896	9.417
836	2626.37	548911.63	698,896	584,277,056	28.914	9.420
837	2629.51	550225.61	700,569	586,376,253	28.931	9.424
838	2632.64	551541.15	702,244	588,480,472	28.948	9.428
839	2635.80	552858.26	703,921	590,589,719	28.965	9.432
840	2638.94	554176.94	705,600	592,704,000	28.983	9.435
841	2642.08	555497.20	707,281	594,823,321	29.000	9.439
842	2645.22	556819.02	708,964	596,947.688	29.017	9.443
843	2648.36	558142.42	710,649	599,077,107	29.034	9.447
844	2651.50	559467.39	712,336	601,211,584	29.052	9.450
845	2654.65	560793.92	714,025	603,351,125	29.069	9.454
846	2657.79	562122.03	715,716	605,495.736	29.086	9.458
847	2660.93	563451.71	717,409	607,645,423	29.103	9.461
848	2664.07	564782.96	719,104	609,800.192	29.120	9.465
849	2667.21	566115.78	720,801	611,960,049	29.138	9.469
850	2670.35	567450.17	722,500	614,125,000	29.155	9.473
851	2673.50	568786.14	724,201	616,295,051	29.172	9.476
852	2676.64	570123.67	725,904	618,470,208	29.189	9.480
853	2679.78	571462.77	727,609	620,650,477	29.206	9.483
854	2682.92	572803.45	729,316	622,835.864	29.223	9.487
855	2686.06	574145.69	731,025	625,026,375	29.240	9.491
856	2689.20	575489.51	732,736	627,222,016	29.257	9.495
857	2692.35	576834.90	734,449	629,422,793	29.274	9.499
858	2695.49	578181.85	736,164	631,628,712	29.292	9.502
859	2698.63	579530.38	737,881	633,839,779	29.309	9.506
860	2701.77	580880.48	739,600	636,056,000	29,326	9.509
861	2704.91	582232.15	741,321	638,277,381	29.343	9.513
862	2708.05	583585.39	743,044	640,503,928	29.360	9.517
863	2711.19	584940.21	744,769	642,735,647	29.377	9.520
864	2714.34	586296.59	746,496	644,972,544	29.394	9.524
865	2717.45	587654.54	748,225	647,214,625	29.411	9.528
866	2720.62	589014.09	749,956	649,461,896	29.428	9.532
867	2723.76	590375.16	751,689	651,714,363	29.445	9.235
868	2726.90	591737.83	753,424	653,972,032	29.462	9.539
869	2730.04	593102.06	755,161	656,234,909	29.479	9.543
870	2733.19	594467.87	765,900	658,503,000	29.496	9.546
871	2736.33	595835.25	758,641	660,776,311	29.513	9.550
872	2739.47	597204.20	760,384	663,054,848	29.529	9.554
873	2742.61	598574.72	762,129	665,338,617	29.546	9.557
874	2745.75	599946.81	763,876	667,627,624	29.563	9.561
875	2748.89	601320.47	765,625	669,921,875	29.580	9.565
876	2752.04	602695.70	767,376	672,221,376	29.597	9.568
877	2755.18	604072.50	769,129	674,526,133	29.614	9.572
878	2758.32	605450.88	770,884	676,836,152	29.631	9.575
879	2761.46	606830.82	772,641	679,151,439	29.648	9.579
880	2764.60	608212.34	774,400	681,472,000	29.665	9.583
881	2767.74	609595.42	776,161	683,797,841	29.682	9.586
882	2770.89	610980.08	777,924	686,128,968	29.698	9.590
883	2774.03	612366.31	779,689	688,465,387	29.715	9.594
884	2777.17	613754.11	781,456	690,807,104	29.732	9.597
885	2780.31	615143.48	783,225	693,154,125	29.749	9.601
886	2783.45	616534.42	784,996	695,506,456	29.766	9.604
887	2786.59	617926.93	786,769	697,864,103	29.782	9.608
888	2789.73	619321.01	788,544	700,227,072	29.799	9.612
889	2792.88	620716.66	790,321	702,595,369	29.816	9.615
890	2796.02	622113.89	792,100	704,969,000	29.833	9.619

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
891	2799.16	623512.68	793,881	707,347,971	29.850	9.623
892	2802.30	624913.04	795,664	709,732,288	29.866	9.626
893	2805.44	626314.98	797,449	712,121,957	29.883	9.630
894	2808.58	627718.49	799,236	714,516,984	29.900	9.633
895	2811.73	629123.56	801,025	716,917,375	29,916	9.637
896	2814.87	630530.21	802,816	719,323,136	29.933	9.640
897	2818.01	631938.43	804.609	721,734,273	29.950	9.644
898	2821.15	633348.22	806,404	724,150,792	29.967	9.648
899	2824.29	634759.58	808,201	726,572,699	29.983	9.651
900	2827.43	636172.51	810,000	729,000,000	30.000	9.655
901	2830.58	637587 01	811,804	721,432,701	30.017	9.658
902	2833.72	639003.09	813,604	733,870,808	30.033	9.662
903	2836.86	640420.73	815,409	736,314,327	30.050	9.666
904	2840.00	641839.95	817,216	738,763,264	30.066	9.669
905	2843.14	643260.73	819,025	741,217,625	30.083	9.673
906	2846.28	644683.09	820,836	743,677,416	30.100	9.676
907	2849 43	646107.01	822,614	746,142,643	30.116	9.680
908	2852.57	647532.51	824,464	748,613,312	30.133	9.683
909	2855.71	648959.58	826,281	751,089,429	30.150	9.687
910	2858.85	650388.21	828,100	753,571,000	30.163	9.690
911	2861.99	651818.43	829,921	756,058,031	30.183	9.694
912	2865.13	653250.21	831,744	758,550,528	30.199	9.698
913	2868.27	654683.56	833,569	761,048,497	30.216	9.701
914	2871.42	656118.48	835,396	763,551,944	30.232	9.705
915	2874.56	657554.98	837,225	766,060,875	30.249	9.708
916	2877.70	658993.04	839,056	768,575,296	30.265	9.712
917	2880.84	660432.68	840,889	771,095,213	30.282	9.715
918	2883.98	661873.88	842.724	773,620,632	30.298	9.718
919	2887.12	663316.66	844,561	776,151,559	30.315	9.722
920	2890.27	664761.01	846,400	778,688,000	30.331	9.726
921	2893.41	666206.92	848,241	781,229,961	30.348	9.729
922	2896.55	667654.41	850,084	783,777,448	30.364	9.733
923	2899.69	669103.47	851.929	786,330,467	30.381	9.736
924	2902.83	670554.10	853,776	788,889,024	30.397	9.740
925	2905.97	672006.30	855,625	791,453,125	30.414	9.743
926	2909.12	673460.08	857,476	794,022,776	30.430	9.747
927	2912.26	674915.42	859,329	796,597,983	30.447	9.750
928	2915.40	676372.33	861,184	799,178,752	30.463	9.754
929	2918.54	677830.82	863,041	801,765,089	30.479	9.757
930	2921.68	679290.87	864,900	804,357,000	30.496	9.761
931	2924.82	680752.50	866,761	806,954,491	30.512	9.764
932	2927.96	682215.69	868,624	809,557.568	30.529	9.768
933	2931.11	683680.46	870,489	812,166,237	30.545	9.771
934	2934.25	685146.80	872,356	814,780,504	30.561	9.775
935	2937.39	686614.71	874,225	817,400,375	30.578	9.778
936	2940.53	688084.19	876,096	820.025,856	30.594	9.783
937	2943.67	689555.24	877,969	822,656,953	30.610	9.785
938	2946.81	691027.86	879,844	825,293,672	30.627	9.789
939	2949.96	692502.05	881,721	827,936,019	30.643	9.792
940	2953.10	693977.82	883,600	830,584,000	30.659	9.796
941	2956.24	695455.15	885,481	833,237,621	30.676	9.799
942	2959.38	696934.06	887,364	835,896,888	30.692	9.803
943	2962.52	698414.53	889,249	838,561,807	30.708	9.806
944	2965.66	699896.58	891,136	841,232,384	30.724	9.810
945	2968.81	701380.28	893,025	843,908,625	30.741	9.813
946	2971.95	702865.38	894,916	846,590,536	30.757	9.817

Number, or Diameter	Circumference.	Circular Area.	Square.	Cube.	Square Root.	Cube Root.
947	2975.09	704352.14	896,809	849,278,123	30.773	9.820
948	2978.23	705840.47	898,704	851,971,392	30.790	9.823
949	2981.37	707330.37	900,601	854,670,349	30.806	9.827
950	2984.51	708821.84	902,500	857,375.000	30.822	9.830
951	2987.66	710314.88	904,401	860.085,351	30.838	9.834
952	2990.80	711809.58	906,304	862,801,408	30.854	9.837
953	2993,94	713305,68	908,209	865,523,177	30.871	9.841
954	2997,08	714803,48	910,116	868,250,664	30.887	9.844
955	3000,22	716302,76	912,025	870,983,875	30.903	9.848
956	3003,36	717803,66	913,936	873,722,816	30.919	9.851
957	3006,50	719306,12	915,849	876,467,493	30.935	9.854
958	3009.65	720810.16	917,764	879,217,912	30.951	9.858
959	3012.79	722315.77	919,681	881,974,079	30.968	9.861
960	3015.93	723822.95	921,600	884,736,000	30.984	9.865
961	3019.07	725331.70	923,521	887.503,681	31.000	9.868
962	3022.21	726842.02	921,444	890,277,128	31.016	9.872
963	3025.35	728353.91	927,369	893,056,347	31.032	9.875
964	3028.50	729867.37	929,296	895,841,344	31.048	9.878
965	3031.64	731382.40	931,225	898,632,125	31.064	9.881
966	3034.78	732899.01	933,156	901,428,696	31.080	9.885
967	3037.92	734417.18	935,089	904,231,063	31.097	9.889
968	3041.06	735936.93	937,024	907,039,232	31.113	9.892
969	3044.20	737458.25	938,961	909,853,203	31.129	9.895
970	3047.35	738981.13	940,900	912,673,000	31.145	9.899
971	3050.49	740505.59	942,841	915,498,611	31.161	9.902
972	3053.63	742031.62	944,784	918,330,048	31.177	9.906
973	3056.77	743559.22	946,729	921,167,317	31.193	9.909
974	3059.91	745088.39	948,676	924,010,424	31.209	9.912
975	3063.05	746619.13	950,625	926,859,375	31.225	9.916
976	3066.19	748151.44	952,576	929,714,176	31.241	9.919
977	3069.34	749685.32	954,529	932,574,833	31.257	9.923
978	3072.48	751220.78	956,484	935,441,352	31.273	9.926
979	3075.62	752757.80	958,441	938,313,739	31.289	9.929
980	3078.76	754296.40	960,400	941,192,000	31.305	9.933
981	3081.90	755836.56	962,361	944,076,141	31.321	9.936
982	3085.04	757378.30	964,324	946,966,168	31.337	9.940
983	3088.19	758921.61	966,289	949,862,087	31.353	9.943
984	3091.33	760466.48	968,256	952,763,904	31.369	9.946
985	3094.47	762012.93	970,225	955,671,625	31.385	9.950
986	3097.61	763560.95	972,196	958,585,256	31.401	9.953
987	3100.75	765110.54	974,169	961,504,803	31.416	9.956
988	3103.89	766661.71	976,144	964,430,272	31.432	9.960
989	3107.04	768214.44	978,121	967,361,669	31.448	9.963
990	3110.18	769768.74	980,100	970,299,000	31.464	9.966
991	3113.32	771324.61	982,081	973,242,271	31.480	9.970
992	3116.46	772882.06	984,064	976,191,488	31.496	9.973
993	3119.60	774441.07	986.049	979,146,657	31.512	9.977
994	3122.74	776001.66	988,036	982,108,784	31.528	9.980
995	3125.89	777563.82	990,025	985,074,875	31.544	9.983
996	3129.03	779127.54	992,016	988,047,936	31.559	9.987
997	3132.17	780692.84	994,009	971,026,973	31.575	9.990
998	3135.31	782259.71	996,004	994,011,992	31.591	9.993
999	3138.45	793828.15	998,001	997,002,999	31.607	9.997
1000	3141.60	785398.16	1,000,000	1,000,000,000	31.623	10.000

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